Try it 1D

1a $x^2+22x=(x+11)^2-11^2$
$=(x+11)^2-121$

1b $2x^2-8x-6=2\left[x^2-4x-3\right]$
$=2\left[(x-2)^2-4-3\right]$
$=2\left[(x-2)^2-7\right]$
$=2(x-2)^2-14$

1c $-x^2+10x=-\left[x^2-10x\right]$
$=-\left[(x-5)^2-25\right]$
$=-(x-5)^2+25$

2a $x^2-3x+1=\left(x-\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+1$
$=\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+1$
$=\left(x-\frac{3}{2}\right)^2-\frac{5}{4}$

So the point $\left(\frac{3}{2},-\frac{5}{4}\right)$ is a minimum

2b $-x^2-7x-12=-\left[x^2+7x+12\right]$
$=-\left[\left(x+\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+12\right]$
$=-\left[\left(x+\frac{7}{2}\right)^2-\frac{49}{4}+12\right]$
$=-\left[\left(x+\frac{7}{2}\right)^2-\frac{1}{4}\right]$
$=-\left(x+\frac{7}{2}\right)^2+\frac{1}{4}$

So the point $\left(-\frac{7}{2},\frac{1}{4}\right)$ is a maximum

2c $2x^2+4x-1=2[x^2+2x-0.5]$
$=2[(x+1)^2-1^2-0.5]$
$=2[(x+1)^2-1-0.5]$
$=2(x+1)^2-3$

So the point (–1, –3) is a minimum

Bridging Exercise 1D

1a $x^2+8x=(x+4)^2-16$

1b $x^2-18x=(x-9)^2-81$

1c $x^2+6x+3=(x+3)^2-9+3$
$=(x+3)^2-6$

1d $x^2+12x-5=(x+6)^2-6^2-5$
$=(x+6)^2-36-5$
$=(x+6)^2-41$

1e $x^2-7x+10=\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+10$
$=\left(x-\frac{7}{2}\right)^2-\frac{9}{4}$

1f $x^2+5x+9=\left(x+\frac{5}{2}\right)^2-\frac{25}{4}+9$
$=\left(x+\frac{5}{2}\right)^2+\frac{11}{4}$

1g $2x^2+8x+4=2\left[x^2+4x+2\right]$
$=2\left[(x+2)^2-4+2\right]$
$=2\left[(x+2)^2-2\right]$
$=2(x+2)^2-4$

1h $3x^2+18x-6=3\left[x^2+6x-2\right]$
$=3\left[(x+3)^2-9-2\right]$
$=3\left[(x+3)^2-11\right]$
$=3(x+2)^2-33$

1i $2x^2-10x+3=2\left[x^2-5x+\frac{3}{2}\right]$
$=2\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{3}{2}\right]$
$=2\left[\left(x-\frac{5}{2}\right)^2-\frac{19}{4}\right]$
$=2\left(x-\frac{5}{2}\right)^2-\frac{19}{2}$

1j $-x^2+12x-1=-\left[x^2-12x+1\right]$
$=-\left[(x-6)^2-36+1\right]$
$=-\left[(x-6)^2-35\right]$
$=-(x-6)^2+35$

1k $-x^2+9x-3=-\left[x^2-9x+3\right]$
$=-\left[\left(x-\frac{9}{2}\right)^2-\frac{81}{4}+3\right]$
$=-\left[\left(x-\frac{9}{2}\right)^2-\frac{69}{4}\right]$
$=-\left(x-\frac{9}{2}\right)^2+\frac{69}{4}$

1l $-2x^2+5x-1=-2\left[x^2-\frac{5}{2}x+\frac{1}{2}\right]$
$=-2\left[\left(x-\frac{5}{4}\right)^2-\frac{25}{16}+\frac{1}{2}\right]$

$$=-2\left[\left(x-\frac{5}{4}\right)^2-\frac{17}{16}\right]$$

$$=-2\left(x-\frac{5}{4}\right)^2+\frac{17}{8}$$

2a $x^2+14x=(x+7)^2-49$

So (–7, –49) is a minimum point

2b $x^2-18x+3=(x-9)^2-81+3$

$$=(x-9)^2-78$$

So (9, –78) is a minimum point

2c $x^2-9x=\left(x-\frac{9}{2}\right)^2-\frac{81}{4}$

So $\left(\frac{9}{2},-\frac{81}{4}\right)$ is a minimum point

2d $-x^2+4x=-\left[x^2-4x\right]$

$$=-\left[(x-2)^2-4\right]$$

$$=-(x-2)^2+4$$

So (2, 4) is a maximum point

2e $x^2+11x+30=\left(x+\frac{11}{2}\right)^2-\frac{121}{4}+30$

$$=\left(x+\frac{11}{2}\right)^2-\frac{1}{4}$$

So $\left(-\frac{11}{2},-\frac{1}{4}\right)$ is a minimum point

2f $-x^2+6x-7=-\left[x^2-6x+7\right]$

$$=-\left[(x-3)^2-9+7\right]$$

$$=-\left[(x-3)^2-2\right]$$

$$=-(x-3)^2+2$$

So (3, 2) is a maximum point

2g $2x^2+16x-5=2\left[x^2+8x-\frac{5}{2}\right]$

$$=2\left[(x+4)^2-16-\frac{5}{2}\right]$$

$$=2\left[(x+4)^2-\frac{37}{2}\right]$$

$$=2(x+4)^2-37$$

So (–4, –37) is a minimum point

2h $-3x^2+15x-2=-3\left[x^2-5x+\frac{2}{3}\right]$

$$=-3\left[\left(x-\frac{5}{2}\right)^2-\frac{25}{4}+\frac{2}{3}\right]$$

$$=-3\left[\left(x-\frac{5}{2}\right)^2-\frac{67}{12}\right]$$

$$=-3\left(x-\frac{5}{2}\right)^2+\frac{67}{4}$$

So $\left(\frac{5}{2},\frac{67}{4}\right)$ is a maximum point.

Try it 1E

1 $7x^2-4x-6=0$

$a=7,\ b=-4,\ c=-6$

$$x=\frac{-(-4)+\sqrt{(-4)^2-4\times7\times(-6)}}{2\times7}$$

$$=1.25$$

$$x=\frac{-(-4)-\sqrt{(-4)^2-4\times7\times(-6)}}{2\times7}$$

$$=-0.68$$

$x=1.25$ or $x=-0.68$

2 $kx^2-x+5=0$

$a=k,\ b=-1,\ c=5$

$b^2-4ac=(-1)^2-4\times k\times5$

$$=1-20k$$

One real solution so $b^2-4ac=0$

So $1-20k=0\Rightarrow k=\frac{1}{20}$

3 $x^2+3x-k=0$

$a=1,\ b=3,\ c=-k$

$b^2-4ac=3^2-4\times1\times(-k)$

$$=9+4k$$

Real solutions so $b^2-4ac\geq0$

So $9+4k\geq0\Rightarrow k\geq-\frac{9}{4}$

4 $kx^2-7x+1=0$

$a=k,\ b=-7,\ c=1$

$b^2-4ac=(-7)^2-4\times k\times1$

$$=49-4k$$

No real solutions so $b^2-4ac<0$

So $49-4k<0\Rightarrow k>\frac{49}{4}$

Bridging Exercise 1E

1a $7x^2+3x-8=0$

$a=7,\ b=3,\ c=-8$

$$x=\frac{-3+\sqrt{3^2-4\times7\times(-8)}}{2\times7}$$

$$=0.88$$