

# St Benedict's Journal of Science

## *6<sup>th</sup> FORM*

## *SPECIAL EDITION*

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# St Benedict's Catholic School

*The Catholic Secondary School for West Suffolk*

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# Welcome to this 6<sup>th</sup> Form Special Edition

The Journal has now entered its 7<sup>th</sup> volume and this first number is a Special Edition intended to showcase examples of the work that 6<sup>th</sup> Form student's produce for their A-Level Sciences.

Featured in this edition are scholarly works in Physics and Chemistry, both theoretical and practical. The students have recorded their sources of information by way of bibliographies, references and in-line citations as they will be required to do at University, or in further academic or scientific life. Their papers are worthy of publication.

In past years 6<sup>th</sup> Form science students have produced work of the highest standard and the Editorial Board, which includes the Head of Science Mr W Stafford, are delighted that the students whose works are published in this edition maintain that high standard.

We congratulate and thank them.

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The **Science Department** also publishes two monthly newsletters: **SCIENCE NEWS *Monthly*** and, for the astronomers, **NIGHT SKY NEWS**.



**Welcome to the October 2022 issue**



These are posted on the school's website and social media accounts.

# 6<sup>TH</sup> FORM

# SPECIAL EDITION

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## **ADRIAN SMITH-DELGADO**

### **PHYSICS REPORT – THE NAVIER STOKES EQUATIONS**

Analysed from two perspectives with an elementary insight into their use within the context of Aeronautics.

#### **Section I – The Perspective of Physics & Aeronautics**

##### **Introduction: The Rise of Flight & A Need to Model Airflow**

Thursday December 17th, 1903, Kitty Hawk, North Carolina.

At the controls of the 1903 Wright Flyer, Orville Wright makes the first manned powered flight in a heavier-than-air aircraft....

44 Years later, on October 14th, 1947: U.S.A.F test pilot Charles Yeager became the first pilot to fly at speeds greater than Mach 1 in his Bell X-1.

Suffice to say, that in less than half a decade the field of aeronautics, and hence aerodynamics, had advanced at a greater rate than most other fields of physics – largely the result of significant investments made by the governments of the belligerents of the First and Second World Wars.

Acknowledging these facts, the reader may be provoked to declare that the Navier Stokes Equations must have been one of the products of ‘the great advance’ of aviation in the early 20th century, yet such a statement is untrue: in fact, the equations predate Orville Wright’s first flight by at least half- a-century.

Therefore, the follow question arises:

***“What was the purpose of the Navier-Stokes Equation before the age of flight?”***

To answer this question, we need to consider the specific discipline of physics that it falls under. Fluid Dynamics.

Fluid Dynamics long predates modern aerodynamics, as suggested by name it is the study of the flow of a medium – early fluid dynamics focused on liquid mediums such as water: however, with the rising interest in lighter-than-air flight in the later 18th century the study of the flow of air around a surface rose in interest, and hence a manner in which to generalise the flow of air around a surface became the objective of numerous physicists and mathematicians.

At this stage it should be noted that advancements in fluid dynamics had been made, David Bernoulli [true to the legacy of the Bernoulli family] had already contributed his ground-breaking Bernoulli’s Principle, which stated that:

***“The pressure a fluid exerts decreases with increasing fluid speed”***

This can be characterised by the following equation:

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$$

[The definition of Bernoulli’s Principle is in fact a derivative of the conservation of energy law, if the reader does not immediately recognise how this is possible: then careful analysis of the equation should reaffirm this statement (note that David Bernoulli did not derive this form of the equation, rather it was the work of famous Swiss mathematician Leonhard Euler).]

Yet there is one problem with Bernoulli's Principle, it assumes that the fluid is incompressible. The reader may wish to remind themselves of elementary particle theory for the states of matter, upon doing so the reader will realise that this renders the principle redundant for air, as it is compressible [it should also be noted that the principle relies on the medium being an ideal fluid, yet ideal fluids are only possible theoretically as no fluids that are both incompressible and have zero viscosity exist (superfluid helium is however one example of a fluid that is extremely close in nature to an ideal fluid)].

Therefore, physicists/mathematicians would need to derive a relationship that could generalise/model the airflow of a viscous and compressible fluid around a surface – such a relationship was eventually derived through the combined work of the Irish physicist Sir George Stokes and the French physicist Claude-Louis Navier during the first half of the 19th century.

We can now introduce one form of the Navier Stokes Equations and outline an important mathematical detail first.

For now, we will analyse the Navier Stokes Equations for an incompressible fluid with a uniform viscosity (this is done as the incompressible fluid forms of the equations is significantly easier to understand and allows for an easy transition to the compressible fluid form) the Navier Stokes Equations for this situation are as follows:

*Note that by considering the incompressible form first, we cannot immediately address the case of air, as air is compressible – the compressible form is introduced later.*

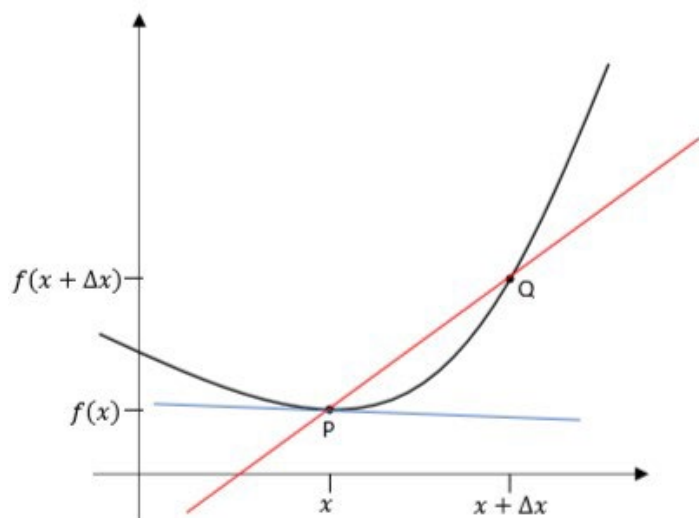
$$\begin{aligned}\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] &= -\frac{\partial P}{\partial x} + \rho g_x + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \\ \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] &= -\frac{\partial P}{\partial y} + \rho g_y + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] \\ \rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] &= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]\end{aligned}$$

It should be noted that all the equations use the  $\partial$  notation, this indicates that all of these equations are partial differential equations, however a reader that is not familiar with multivariable calculus may not immediately understand why the Navier Stokes equations are partial differential equations. To understand why partial differentials are used we must consider the nature of differentiation, elementary calculus gives us the following relationship that defines a differential:

$$\lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = f'(x)$$

For the sake of a reader not well-versed in calculus, we can picture this graphically:

In the adjacent diagram, the plot of the arbitrary function  $f(x)$  is given, two points on the curve are given, a fixed-point P given by the function at  $x$  and a point q which is free to move around given by the function at  $x + \Delta x$ .





A red line connecting P and Q, is called the secant line: the reader should note that as  $\Delta x \rightarrow 0$  point Q tends towards point P, hence our secant line becomes increasingly similar to the blue tangent line representing the gradient at point P. By considering the gradient of the secant line at infinitesimally small values the gradient at P is found [this is an elementary explanation of limits, however it should be noted that despite  $\Delta x \rightarrow 0$ ,  $\Delta x \neq 0$ ].

From this the reader can ascertain the following:

**Differentiation is measure of the change in the output of a function relative to a given change in the value of the variable.**

Yet, what about in cases where a function has more than one variable? Many relationships used in the practical world are not reliant on a single variable.

Let's consider the following function:

$$f(x,y) = \sin(x)\cos(y)$$

Note that it is only possible to plot this using three-dimensions: x,y and z.

**[PICTURED TO THE RIGHT]**

At this stage the reader may feel quite overwhelmed if asked how it is possible to measure the change in output of the function when one variable is changed by a given amount.

However, this can be done rather simply: we differentiate the function with respect to one variable at any given time – the rest are treated as constants: this allows us to accurately measure the effect a change in one variable has on the function as a whole.

Therefore, we can partially differentiate this function with respect to either x or y:

**Partial Derivative with respect to x:**

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} (\sin(x) \cos(y))$$

*Recall that y is treated as a constant, hence cos y must also be a constant*

$$\therefore \frac{\partial}{\partial x} (\sin(x) \cos(y)) = \cos(y) \cdot \frac{\partial}{\partial x} (\sin(x)) = \cos(y) \cos(x)$$

**Partial Derivative with respect to y:**

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y} (\sin(x) \cos(y))$$

*Recall that x is treated as a constant, hence sin x must also be a constant*

$$\therefore \frac{\partial}{\partial y} (\sin(x) \cos(y)) = \sin(x) \cdot \frac{\partial}{\partial y} (\cos(y)) = \sin(x) \cdot -\sin(y) = -\sin(x) \sin(y)$$

Therefore, the Navier Stokes Equations are partial differential equations as this allows us to consider the effects of one variable on the result independent of other variables [the reasoning for partial differential equations can also be considered if we recall that we have three different equations each relating to the x,y and z components of flow: the reader should recall that elementary vector theory dictates that such components should be considered independent of one another].

*[It should be noted that this section involved simplified elementary multivariable calculus for the sake of clarity, and also to prevent the author from digressing due to a significant personal interest in multivariable calculus.]*

Now that we have clarified perhaps the most important detail so far, we can begin to explore the Navier Stokes Equations with a physics-based perspective.

### Similarities to Newton's Second Law

Let's recall our three equations:

$$\begin{aligned}\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] &= -\frac{\partial P}{\partial x} + \rho g_x + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \\ \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] &= -\frac{\partial P}{\partial y} + \rho g_y + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] \\ \rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] &= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]\end{aligned}$$

First it may help to combine all of these equations into one equation, by combining the components of velocity into the vector quantity  $\vec{v}$  :

$$\rho \left[ \frac{D\vec{v}}{Dt} \right] = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{v}$$

The reader will notice two key differences: the use of  $\left[ \frac{D\vec{v}}{Dt} \right]$  notation and the use of nabla notation  $[\nabla]$ , both will be discussed in greater detail in section II of this paper.

At this stage the reader should be made aware of the fact that the Navier Stokes equation is applied on a per-unit-volume basis, that is to say that the use of rho to represent density effectively implies mass. What the reader should also notice is that the differential coefficient of a function that gives velocity gives acceleration, hence the reader should begin to note a similarity to Newton's Second Law.

Recall Newton's Second Law:

**The resultant force acting on a body is directly proportional to the rate at which its velocity is changing.**

This can be expressed as:  $F = m \cdot \frac{v-u}{t} = m \cdot \frac{\Delta v}{t} = ma$

And our general Navier Stokes Equation:

$$\rho \left[ \frac{D\vec{v}}{Dt} \right] = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{v}$$

Therefore, let us postulate the following:

$\rho$  – Mass term

$\frac{D\vec{v}}{Dt}$  – Acceleration Term

$-\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{v}$  – Resultant Force Term

The first two postulations should be readily accepted by the reader, as they have been explained previously: however, the 'Resultant Force Term' may not be as clear. Therefore, this term must be explained before we can continue.

### The Resultant Force Term

We begin by dividing the Resultant Force Term into 3 separate sub-terms, each sub-term characterising an external force (two of which are surface forces and the third a body force):

$$F_r = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{v}$$

The  $-\nabla P$  sub-term is known as the pressure gradient term.

First it may help to understand the term pressure gradient, a pressure gradient is a measure of the rate of change of pressure. Recall that pressure is given by  $P = F/A$ , hence a change in pressure results in a change in the magnitude of the force which, by use of Newton's Laws, results in a body accelerating: this is why fluids [typically] flow from areas of high pressure to areas of low pressure. The pressure gradient term is measure of the force applied by a change of pressure as a fluid flows.

The reader should note that this term is negative, this is because the direction of increasing pressure is directly opposite to the direction of flow: since as fluid flows from high pressure to low pressure it accelerates according to Bernoulli's Principle and the pressure gradient term was positive it would imply that fluid is also accelerating when flowing from low pressure to high pressure which is a contradiction: hence why the negative sign is used on the term.

Note that this term will be a vector term, as the pressure gradient is calculated from a 3-dimensional space of pressure.

**The  $pg$  sub-term is known as the body force term.**

This is the simplest of the three terms to understand, unlike contact/surface forces a body force acts upon the complete volume of a body rather than a surface. Body forces are usually the result of gravitational, electrical, or magnetic fields – however in the case we are considering only the former is applicable.

We know that  $W = mg$ , and also that we are currently utilising a per-unit-volume system, therefore we can consider body force  $F_b$  as effectively equal to the weight acting on a body in our system:

$$\Rightarrow F_b = W = mg = pg \text{ [For a per unit volume system]}$$

This term considers the effects of gravity on the resultant force, it is important to note that gravitational acceleration  $g$  in the Navier Stokes equation is given as a vector.

**The  $\mu \nabla^2 u$  is known as the diffusion term.**

This is perhaps the most difficult of the three terms to understand, perhaps it is best to define the term simply, as attempting to do so in a more complex manner would lead to a large digression from the main subject of this section due to the complex nature of explaining Newtonian fluids, the diffusion term is a measure of the viscous force in a fluid: that is to say the force caused by interactions in the fluid due to shear stress.

Now with the link to Newton's Second Law made clear, this section can be concluded by explaining the uses of the Navier Stokes Equations in the aeronautics industry, and thus leaving various complex mathematical concepts that are involved in the Equations to be explained in the second section.

## Applications of the Navier Stokes Equations in Aeronautics

First, we need to consider the medium aeronautics considers air, which is a compressible medium. Therefore, we cannot work with the Navier Stokes Equation we used before, rather we need to consider one of its other forms:

The incompressible fluid Navier Stokes Equation is given as:

$$\rho \left[ \frac{D\vec{v}}{Dt} \right] = -\nabla P + pg + \mu \nabla^2 \vec{v}$$

Therefore, we can now discuss the uses of the Navier Stokes Equations....



Engineers and Physicists are interested in the Navier Stokes Equations in the context of aeronautics as they allow them to model airflow around a surface in a purely theoretical manner – although the Navier Stokes Equations are difficult to solve due to their mathematical properties [hence why the equations are extremely interesting to mathematicians, and thus why Section II constitutes such a large part of the paper], physicists/engineers are able to make approximate solutions which allow them to relatively accurately model airflow.

In terms of aeronautics this allows engineers to model the airflow around critical components, such as the wings and engines [Rainer, 2015], which in turn allows them to minimise drag and improve flight stability without the cost and risk of designing and flying numerous prototypes.

Suffice to say, the Navier Stokes Equations are a critical component in modern aeronautics....

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## Section II – The Perspective of Mathematics

The results of the Navier-Stokes Equations are highly useful to physicists and engineers, whereas mathematicians tend to focus-on and appreciate the complexity and unique properties of the equations. In this section the complex nature of the Navier Stokes Equations is examined at a relatively elementary level from a mathematical perspective, however we first need to elaborate on some of the physics.

### Basic Concepts

Before we begin to analyse the Navier Stokes Equations from a mathematical perspective, we need to consider some key concepts.

We begin by discussing velocity vector fields, the Navier Stokes equation considers an infinitesimal volume of fluid within a velocity vector field, that is to say a field where all points in space are assigned a velocity vector  $\vec{v}$  – therefore this infinitesimal volume of fluid has a velocity vector  $\vec{v}$  that is a function of their position  $\vec{x} = \{x,y,z\}$  and time  $t$ , position  $\vec{x}$  is also a function of time  $t$ .

This allows us to consider the divergence of the velocity vector  $\nabla \cdot \vec{v}$  later in our continuity equation.

The other concept we need to consider is that of divergence, the divergence in vector fields is a measure of the extent to which the vectors in a field are diverted away from an infinitesimal volume at a point.

### Analysing the LHS in greater detail

When presented with the Navier Stokes Equations in the previous section, it was easy to dismiss the left-hand side as the same product of two variables as is found in Newton's 2nd Law. However, in doing so we dismissed any further elaborate on the 'acceleration' term, as well as the mathematical challenge it poses.

Recall the equations:

$$\begin{aligned}\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] &= -\frac{\partial P}{\partial x} + \rho g_x + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \\ \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] &= -\frac{\partial P}{\partial y} + \rho g_y + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] \\ \rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] &= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right]\end{aligned}$$

Focusing on the LHS...

$$\begin{aligned} & \rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] \\ & \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] \\ & \rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] \end{aligned}$$

Removing the  $\rho$  from each expression...

$$\begin{aligned} & \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] \\ & \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] \\ & \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] \end{aligned}$$

Therefore, let's consider the following: if these equation represent acceleration in their respective dimensions, we can ascertain the following [recall that we are considering an infinitesimally small volume of fluid]:

*$x, y, z$  are functions of time representing displacement*

*in their respective dimension/component*

*therefore  $u, v, w$  must be the  $x, y$  and  $z$  components respectively of the velocity of flow of a fluid  $\vec{v}$  at a point defined by  $(x, y, z)$  and time  $t$*

*therefore  $u, v, w$  must all be the results of multivariable functions of  $x, y, z, t$*

$$\begin{aligned} \Rightarrow a_x &= \frac{\delta u}{\delta t} \\ \Rightarrow a_y &= \frac{\delta v}{\delta t} \\ \Rightarrow a_z &= \frac{\delta w}{\delta t} \end{aligned}$$

[Lagrangian time derivatives are discussed later in this section; however, the reader should assume the following to be true:]

$$\begin{aligned} \therefore a_x &= \frac{Du}{Dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot \frac{\delta x}{\delta t} + \frac{\partial u}{\partial y} \cdot \frac{\delta y}{\delta t} + \frac{\partial u}{\partial z} \cdot \frac{\delta z}{\delta t} \\ \therefore a_y &= \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \cdot \frac{\delta x}{\delta t} + \frac{\partial v}{\partial y} \cdot \frac{\delta y}{\delta t} + \frac{\partial v}{\partial z} \cdot \frac{\delta z}{\delta t} \\ \therefore a_z &= \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \cdot \frac{\delta x}{\delta t} + \frac{\partial w}{\partial y} \cdot \frac{\delta y}{\delta t} + \frac{\partial w}{\partial z} \cdot \frac{\delta z}{\delta t} \end{aligned}$$

Upon closer inspection of these equations and our previous statements, it must be true that:

$$\frac{\delta x}{\delta t} = u, \frac{\delta y}{\delta t} = v, \frac{\delta z}{\delta t} = w$$

Therefore,

$$\begin{aligned}\therefore a_x &= \frac{Du}{Dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot \frac{\delta x}{\delta t} + \frac{\partial u}{\partial y} \cdot \frac{\delta y}{\delta t} + \frac{\partial u}{\partial z} \cdot \frac{\delta z}{\delta t} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial y} \cdot v + \frac{\partial u}{\partial z} \cdot w \\ \therefore a_y &= \frac{Dv}{Dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \cdot \frac{\delta x}{\delta t} + \frac{\partial v}{\partial y} \cdot \frac{\delta y}{\delta t} + \frac{\partial v}{\partial z} \cdot \frac{\delta z}{\delta t} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x} \cdot u + \frac{\partial v}{\partial y} \cdot v + \frac{\partial v}{\partial z} \cdot w \\ \therefore a_z &= \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \cdot \frac{\delta x}{\delta t} + \frac{\partial w}{\partial y} \cdot \frac{\delta y}{\delta t} + \frac{\partial w}{\partial z} \cdot \frac{\delta z}{\delta t} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \cdot u + \frac{\partial w}{\partial y} \cdot v + \frac{\partial w}{\partial z} \cdot w\end{aligned}$$

Therefore, we return to the acceleration terms we began with:

$$\begin{aligned}&\left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] \\ &\left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] \\ &\left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right]\end{aligned}$$

At this stage the reader should have established a firm understanding of the significance of the LHS of the equation, however there may still be some uncertainty as to why there are 2 distinctive forms for the sub-terms involved in the acceleration terms.

Recall the expression for the x-component of the acceleration of an infinitesimally small volume of fluid:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

Now note the two distinctive groups of sub-terms:

$$\frac{\partial u}{\partial t} \text{ and } u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

These two groups describe two different types of acceleration: local acceleration and convective acceleration respectively.

**Local Acceleration** considers the change in velocity of a fluid relative to a fixed point, the value of  $\partial u / \partial t$  is dependent on the type of flow.

- Under steady flow, the velocity of the fluid at any given point is uniform, hence  $\partial u / \partial t$  is always equal to 0.
- Under unsteady flow, the velocity of the fluid at any given point is non-uniform, hence  $\partial u / \partial t$  is always non-zero.

**Convective Acceleration** considers the change in velocity of an infinitesimally small volume of fluid as a result of interactions with the environment/its position in a field of space: notably interactions with other infinitesimally small volumes of fluid and container walls, if we consider velocity to always be positive in the direction of flow, then the  $u, v, w$  are always greater than or equal to 0 whereas the  $\partial u / \partial x$ ,  $\partial u / \partial y$ , ...,  $\partial w / \partial z$  can take any real value.

### Conservation of Mass, Momentum & Continuity Equations

At this stage of the paper, the reader must be made aware of the fact that the Navier Stokes Equations are a set of continuity equations – and that we have only analysed one set of those equations: the ones relating to momentum.

Therefore, it is imperative that we analyse the other equation that forms the Navier Stokes Equations.

## The Continuity Equation

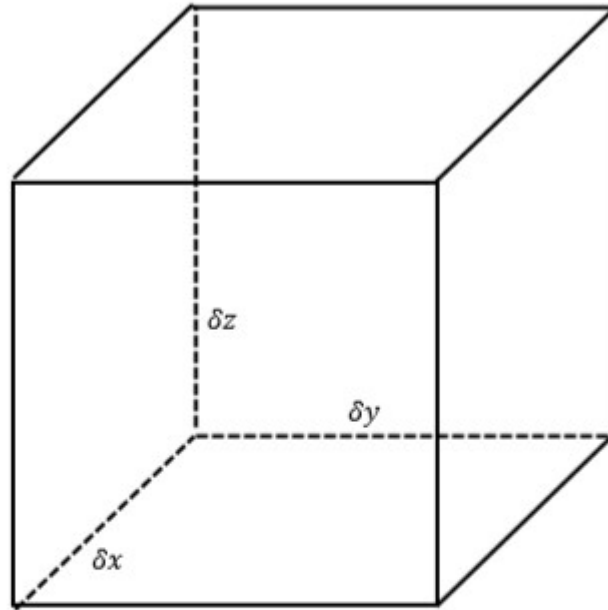
First, we must define a continuity equation, in rather simple terms a continuity equation is the fluid equivalent of the Law of Conservation of Mass.

**Therefore, we can now attempt to derive the continuity equation:**

*Consider an infinitesimal cube through which fluids travels.*

*Consider the cube axes x;y;z, and now let each cube length equal  $\delta x, \delta y, \delta z$  respectively.*

*Drawing a diagram we obtain the following:*



*Consider the mass of fluid entering and leaving the cube,  
assign the variables  $m_{in}$  and  $m_{out}$  respectively to these values.*

$$\therefore \text{The mass flow rate} = \frac{\delta m}{\delta t} = m'$$

*In fluid dynamics the formal notation for the mass flow rate is not  $m'$ , but  $\dot{m}$ .*

[Rather interestingly, this choice of notation is the result of  $m'$  being a case of Lagrange's notation for the differential of a function/variable. Physicists prefer Newton's notation which involves dots directly above a variable, the number of dots indicates the degree of the differential.]

*Consider a possible relationship between  $\dot{m}_{in}$  and  $\dot{m}_{out}$ .*

*The following form should be intuitive.*

$$\Rightarrow \dot{m}_{in} = \dot{m}_{out} + \frac{\delta m}{\delta t} \text{ [web.mit.edu, n.d]}$$

*This is perhaps better understood in the following form:*

$$\dot{m}_{in} - \dot{m}_{out} = \frac{\delta m}{\delta t}$$

*Consider the definition of  $\dot{m}_{in}$  and  $\dot{m}_{out}$ , they are mass flow rate in and out of our cube*

*(The volume of this infinitesimal cube is known as the control volume.)*

*Mass flow rate is defined as the change in mass divided by the change in time.*

$$\therefore \dot{m} = \frac{\delta m}{\delta t}$$

However since we know that  $m = \rho \cdot V$ , we can calculate the volumetric flow rate and multiply by  $\rho$

$$\Rightarrow \dot{m} = \frac{\delta m}{\delta t} = \rho \cdot \dot{V}$$

Now consider the definition of the volumetric flow rate.

Therefore we need to consider the volume of fluid that flows in a given time interval

This is easily done by considering the cross sectional area of the flow,

and multiplying by flow velocity

Therefore if we have a velocity of  $x$ , and a cross sectional area of  $yz$ , then volumetric flow rate is  $xyz$  as in one unit of time the fluid travels  $x$  distance, giving us the third dimension to consider alongside cross sectional area, hence giving us volume per unit time – volumetric flow rate.

$$\Rightarrow \dot{m} = \frac{\delta m}{\delta t} = \rho \cdot \dot{V} = \rho \cdot v \cdot A$$

Now consider  $m$  and  $\frac{\delta m}{\delta t}$ ,  $\partial$  is used as we must take a partial derivative as  $m$  is taken from a multivariable function.

$$m = \rho V \Rightarrow \frac{\partial m}{\partial t} = \frac{\partial}{\partial t}(\rho V)$$

Recall that  $V = \delta x \cdot \delta y \cdot \delta z = \delta x \delta y \delta z$

$$\therefore \frac{\partial m}{\partial t} = \frac{\partial}{\partial t}(\rho V) = \frac{\partial}{\partial t}(\rho \cdot \delta x \delta y \delta z)$$

Now we can apply this to our control volume:

Assume that fluid flows in the positive  $x, y$  and  $z$  directions.

**First, we calculate the mass flow rate into the cube.**

Recall that in the previous sub – section we defined variables  $u, v, w$

as the components of the velocity of a fluid.

Consider the fluid flowing in the positive  $x$  direction at a flow velocity  $u_x$  – recall that we must consider components due to having a 3 Dimensional system.

$$\text{Recall that: } \dot{m} = \frac{\delta m}{\delta t} = \rho \cdot \dot{V} = \rho \cdot v \cdot A$$

$$\therefore \dot{m}_{in_x} = \rho \cdot v \cdot A = \rho \cdot u_x \cdot [\delta z \cdot \delta y] = \rho u_x \delta z \delta y$$

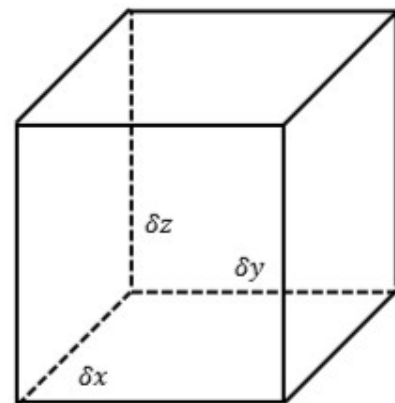
Now we calculate the mass flow rate into the cube for the flow in the positive  $y$  and  $z$  direction.

The fluid flowing in the positive  $y$  and  $z$  directions must have

the flow velocities  $v_y$  and  $w_z$  respectively.

$$\therefore \dot{m}_{in_y} = \rho \cdot v \cdot A = \rho \cdot v_y \cdot [\delta x \cdot \delta z] = \rho v_y \delta x \delta z$$

$$\therefore \dot{m}_{in_z} = \rho \cdot v \cdot A = \rho \cdot w_z \cdot [\delta x \cdot \delta y] = \rho w_z \delta x \delta y$$



**Now, we calculate the mass flow rate out of the cube.**

First, we consider the fluid [component] velocities at the exit faces, again we assume that fluid overall flows in the positive x, y and z directions. Therefore, we assign the following variables to these components:

$$u_{x+\delta x}, v_{y+\delta y} \text{ and } w_{z+\delta z}$$

$$\therefore \dot{m}_{out_x} = \rho \cdot v \cdot A = \rho \cdot u_{x+\delta x} \cdot [\delta z \cdot \delta y] = \rho u_{x+\delta x} \delta z \delta y$$

$$\therefore \dot{m}_{out_y} = \rho \cdot v \cdot A = \rho \cdot v_{y+\delta y} \cdot [\delta x \cdot \delta z] = \rho v_{y+\delta y} \delta x \delta z$$

$$\therefore \dot{m}_{out_z} = \rho \cdot v \cdot A = \rho \cdot w_{z+\delta z} \cdot [\delta x \cdot \delta y] = \rho w_{z+\delta z} \delta x \delta y$$

**Therefore, we can now collect these values, to give us an overall value for mass flow rates in and out of the control volume:**

$$\therefore \dot{m}_{in} = \rho u_x \delta z \delta y + \rho v_y \delta x \delta z + \rho w_z \delta x \delta y$$

$$\therefore \dot{m}_{out} = \rho u_{x+\delta x} \delta z \delta y + \rho v_{y+\delta y} \delta x \delta z + \rho w_{z+\delta z} \delta x \delta y$$

$$\therefore \dot{m}_{in} - \dot{m}_{out} = \frac{\delta m}{\delta t}$$

$$\Rightarrow (\rho u_x \delta z \delta y + \rho v_y \delta x \delta z + \rho w_z \delta x \delta y) - (\rho u_{x+\delta x} \delta z \delta y + \rho v_{y+\delta y} \delta x \delta z + \rho w_{z+\delta z} \delta x \delta y) = \frac{\delta m}{\delta t}$$

$$\Rightarrow (\rho u_x \delta z \delta y + \rho v_y \delta x \delta z + \rho w_z \delta x \delta y) - (\rho u_{x+\delta x} \delta z \delta y + \rho v_{y+\delta y} \delta x \delta z + \rho w_{z+\delta z} \delta x \delta y) = \frac{\partial}{\partial t} (\rho \cdot \delta x \delta y \delta z)$$

$$\text{Recall the property of differentials, where } a \text{ is a constant: } \frac{\partial}{\partial x} (ax) = a \cdot \frac{\partial}{\partial x} (x)$$

$$\Leftrightarrow (\rho u_x \delta z \delta y + \rho v_y \delta x \delta z + \rho w_z \delta x \delta y) - (\rho u_{x+\delta x} \delta z \delta y + \rho v_{y+\delta y} \delta x \delta z + \rho w_{z+\delta z} \delta x \delta y) = \delta x \delta y \delta z \cdot \frac{\partial}{\partial t} (\rho)$$

$$\Rightarrow \left( \frac{\rho u_x}{\delta x} + \frac{\rho v_y}{\delta y} + \frac{\rho w_z}{\delta z} \right) - \left( \frac{\rho u_{x+\delta x}}{\delta x} + \frac{\rho v_{y+\delta y}}{\delta y} + \frac{\rho w_{z+\delta z}}{\delta z} \right) = \frac{\partial}{\partial t} (\rho)$$

$$\Leftrightarrow \left( \frac{\rho u_x - \rho u_{x+\delta x}}{\delta x} \right) + \left( \frac{\rho v_y - \rho v_{y+\delta y}}{\delta y} \right) + \left( \frac{\rho w_z - \rho w_{z+\delta z}}{\delta z} \right) = \frac{\partial}{\partial t} (\rho)$$

At this stage the reader should notice that the terms on the LHS have a very similar form to the definition of a derivative [discussed earlier in the paper]. Hence we can apply the following.

$$\therefore \frac{\rho u_x - \rho u_{x+\delta x}}{\delta x} = - \left( \frac{-\rho u_x + \rho u_{x+\delta x}}{\delta x} \right) = - \left( \frac{\rho u_{x+\delta x} - \rho u_x}{\delta x} \right)$$

Now recall that our control volume is infinitesimal, hence  $x \rightarrow 0$ ,  $y \rightarrow 0$  and  $z \rightarrow 0$ .

Hence we effectively have:

$$- \left( \frac{\rho u_{x+\delta x} - \rho u_x}{\delta x} \right) - \left( \frac{\rho v_{y+\delta y} - \rho v_y}{\delta y} \right) - \left( \frac{\rho w_{z+\delta z} - \rho w_z}{\delta z} \right) = \frac{\partial}{\partial t} (\rho)$$

$$\therefore \left( \lim_{x \rightarrow 0} - \left( \frac{\rho u_{x+\delta x} - \rho u_x}{\delta x} \right) \right) + \left( \lim_{y \rightarrow 0} - \left( \frac{\rho v_{y+\delta y} - \rho v_y}{\delta y} \right) \right) + \left( \lim_{z \rightarrow 0} - \left( \frac{\rho w_{z+\delta z} - \rho w_z}{\delta z} \right) \right) = \frac{\partial}{\partial t} (\rho)$$

We can rewrite the LHS as three terms of partial derivatives:

$$- \frac{\partial}{\partial x} (\rho u) - \frac{\partial}{\partial y} (\rho v) - \frac{\partial}{\partial z} (\rho w) = \frac{\partial}{\partial t} (\rho)$$

Altering the form slightly gives us the continuity equation for the Navier Stokes Equations:

$$\frac{\partial}{\partial t} (\rho) + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0 \blacksquare$$



This equation allows for the generalisation of the conservation of mass in a fluid, when solved simultaneously with the conservation of momentum equations show previously, the flow of fluid can be generalised.

It should be noted that the continuity equation is often written in the shortened form:

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0$$

This form is very important to the aeronautical context we consider in this paper, that is because if  $\nabla \cdot (\rho \vec{v}) \neq 0$  it implies that  $\partial_t \rho \neq 0$  hence density of the fluid varies hence it is compressible. Such a case makes solving the Navier Stokes Equations for compressible fluids incredibly difficult.

[Incompressible fluids are fluids where  $\partial_t \rho = 0$  therefore rendering  $\nabla \cdot (\rho \vec{v}) = 0$  to be significantly easier to solve.]

### Use of $\nabla$ - Nabla Notation

Recall our equation for incompressible flow:

$$\rho \left[ \frac{D\vec{v}}{Dt} \right] = -\nabla P + \rho g + \mu \nabla^2 \vec{v}$$

Recall that we expanded the LHS to:

$$\begin{aligned} & \rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] \\ & \rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] \\ & \rho \left[ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] \end{aligned}$$

And that we could split the acceleration terms into two subgroups, local and convective:

$$\begin{aligned} & \therefore \rho \left[ \frac{\partial u}{\partial t} \right] + \rho \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] \\ & \therefore \rho \left[ \frac{\partial v}{\partial t} \right] + \rho \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] \\ & \therefore \rho \left[ \frac{\partial w}{\partial t} \right] + \rho \left[ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] \end{aligned}$$

We can combine component terms by considering the general [vector] variable for flow velocity:  $\vec{v}$

$$\rho \left[ \frac{\partial \vec{v}}{\partial t} \right] + \rho [(\vec{v} \cdot \nabla) \vec{v}]$$

At this stage the reader is still unfamiliar with the  $\nabla$  notation, hence this subsection is devoted to explaining its purpose. In simple terms, mathematical speaking, the nabla notation represents the vector with [partial] differential components relating to their respective component, for example:

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix}$$

It should be noted that nabla notation raised to a degree gives the degree of the differential, for example:

$$\nabla^n = \begin{pmatrix} \frac{\partial^n}{\partial x^n} \\ \frac{\partial^n}{\partial y^n} \\ \frac{\partial^n}{\partial z^n} \end{pmatrix} = \begin{pmatrix} \partial_x^n \\ \partial_y^n \\ \partial_z^n \end{pmatrix}$$

The reader should also note the “.” notation, when discussing vector quantities this is known as the scalar product. I.e.,

$$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Therefore:

$$\begin{aligned} (\vec{v} \cdot \nabla) &= \begin{pmatrix} u \\ v \\ w \end{pmatrix} \cdot \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} = u \partial_x + v \partial_y + w \partial_z \\ \therefore (\vec{v} \cdot \nabla) \vec{v} &= (u \partial_x + v \partial_y + w \partial_z) \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} (u \partial_x + v \partial_y + w \partial_z) u \\ (u \partial_x + v \partial_y + w \partial_z) v \\ (u \partial_x + v \partial_y + w \partial_z) w \end{pmatrix} \\ &= \begin{pmatrix} u [\partial_x(u)] + v [\partial_y(u)] + w [\partial_z(u)] \\ u [\partial_x(v)] + v [\partial_y(v)] + w [\partial_z(v)] \\ u [\partial_x(w)] + v [\partial_y(w)] + w [\partial_z(w)] \end{pmatrix} \end{aligned}$$

The reader should note that this is identical to the component forms for convective acceleration above.

Therefore, our new general incompressible Navier Stokes Equation is:

$$\rho \left[ \frac{\partial \vec{v}}{\partial t} \right] + \rho [(\vec{v} \cdot \nabla) \vec{v}] = -\nabla P + \rho \mathbf{g} + \mu \nabla^2 \vec{v}$$

## Lagrangian Time Derivatives

By this stage the reader will have noticed the use of  $D/Dt$  notation, specifically in the subsection analysing the LHS of the Navier Stokes Equations in greater detail. It is similar to the  $\delta/\delta t$  notation, however both represent very distinct derivatives.

The former represents the Lagrangian time derivative, whereas the latter represents the Eulerian time derivative.

Both define the rate of change of a quantity within a fluid, however, a Eulerian time derivative does so relative to a fixed point in a field whereas a Lagrangian time derivative does so relative to a nonfixed-point in a field.

The two derivatives however can be united by means of a simple relationship.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

The reader should note a clear link between the local acceleration and the Eulerian time derivative and a link between the convective acceleration and the Lagrangian time derivative.

## Basis of the Diffusion term and the Pressure Gradient term.

It is easiest to begin by analysing the diffusion term for an incompressible fluid where  $\nabla \cdot \vec{v} = 0$ .

$$\therefore F_{viscosity} = \mu \nabla^2 \vec{v}$$

$$F_{pressure} = -\nabla P$$

We begin by first considering our infinitesimal control volume of fluid from the previous subsections:

Consider the following stresses acting at normal to the faces parallel to the x-z plane.

Consider the forces acting against/in the y-direction, note the red arrow denoting stress in the positive y direction and the blue arrow denoting stress in the negative y direction, consider the fact that the former acts from position  $y + \delta y$  on the y-axis and the latter acts from position  $y$  on the y-axis [the reader should recall that use of components is inevitable as vector values are being used].

Doing the same in the positive/negative x and z directions gives the following stresses (standard notation for normal stress is lowercase sigma  $\sigma$ ):

$$\sigma_{xx}(x) \text{ and } \sigma_{xx}(x + \delta x) [x \text{ component}]$$

$$\sigma_{yy}(y) \text{ and } \sigma_{yy}(y + \delta y) [y \text{ component}]$$

$$\sigma_{zz}(z) \text{ and } \sigma_{zz}(z + \delta z) [z \text{ component}]$$

Consider the force of internal pressure, which by convention always act at normal to a surface, such a force is denoted as  $-P$  and must be considered alongside out non-shear stress cases.

Now we consider the shear stresses acting against/in the y-direction:

The blue arrows indicate shear stresses acting against/in the y direction found in the  $z - y$  and  $\delta z - \delta y$  planes. Whereas the red arrows indicate shear stresses acting against/in the y-direction found in the  $x - y$  and  $\delta x - \delta y$  planes.

Therefore, the shear stresses are as follows [where  $\tau_{ij}$   $i$  indicates the axis at the normal to the plane in which the stress acts and  $j$  indicates the direction in which the stress acts.]

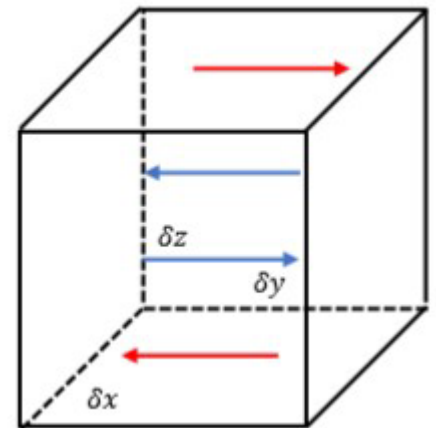
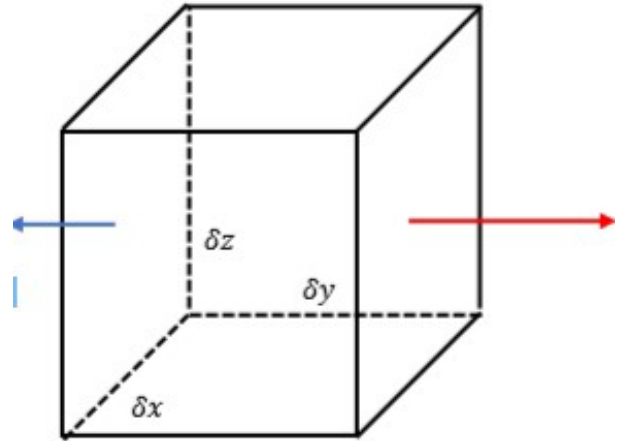
Considering the stresses which act when considering each component, we obtain the following:

$$\tau_{yx}(y) \mid \tau_{yx}(y + \delta y) \mid \tau_{zx}(z) \mid \tau_{zx}(z + \delta z) [x \text{ component}]$$

$$\tau_{xy}(x) \mid \tau_{xy}(x + \delta x) \mid \tau_{zy}(z) \mid \tau_{zy}(z + \delta z) [y \text{ component}]$$

$$\tau_{xz}(x) \mid \tau_{xz}(x + \delta x) \mid \tau_{yz}(y) \mid \tau_{yz}(y + \delta y) [z \text{ component}]$$

[Note that we use the  $(x)$  and  $(x + \delta x)$  in the diagrams only, they are not used in what follows, as we are considering the resultant stress.]



After listing all stresses acting on the control volume, we introduce the concept of the stress tensor denoted by  $\sigma_{ij}$ , it is a form of representing all the stresses acting on our control volume.

$$\text{Defined by } \sigma_{ij} = -P\delta_{ij} + \sigma'_{ij}$$

This gives a result in matrix form of:

$$\sigma_{ij} = \begin{pmatrix} -P + \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -P + \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -P + \sigma_{zz} \end{pmatrix}$$

[The reader may note that the  $-P\delta_{ij}$  term is not present in the elements of the matrix where  $i \neq j$ , this is due to the  $\delta_{ij}$  operator, referred to as the Kronecker delta, it always has a non-zero value except for cases where  $i = j$  – this is to allow for the pressure term to be considered only in nonshear cases where stress is acting at a normal.]

The stress tensor can be considered as the sum of two tensors, the Hydrostatic Stress Tensor, and the Deviatoric Stress Tensor:

*Hydrostatic Stress Tensor:*

$$\text{Defined by } -P\delta_{ij}$$

$$-\begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

*Deviatoric Stress Tensor:*

*Note the inclusion of normal and shear stresses*

$$\sigma'_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

Hence the reader should readily accept that:

$$\sigma_{ij} = \begin{pmatrix} -P + \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -P + \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -P + \sigma_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} - \begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix}$$

We should note that in the Navier Stokes Equation, the diffusion and pressure gradient terms are actually taken by using Nabla notation to measure the divergence of the stress tensor:

This is derived from the Cauchy momentum equation:  $\rho \frac{D\vec{v}}{Dt} = \nabla \cdot \sigma + f$

Hence:

$$\nabla \cdot \sigma_{ij} = \nabla \cdot \begin{pmatrix} -P + \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -P + \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -P + \sigma_{zz} \end{pmatrix} = \nabla \cdot \left[ \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} - \begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix} \right]$$

We can however make an important manipulation, note the form of the hydrostatic stress tensor, it is identical to the identity vector multiplied by the scalar factor  $p$ , hence:

$$\begin{aligned} \therefore &= \nabla \cdot \left[ \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} - \begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix} \right] = \nabla \cdot \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} - \nabla \cdot P \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \nabla \cdot \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} - \nabla \cdot PI \end{aligned}$$

Which, if we consider our vector form of the Navier Stokes Equation gives us the following:

$$\begin{aligned} &= \nabla \cdot \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} - \nabla P \\ &[= \nabla \cdot \sigma' - \nabla P] \end{aligned}$$

Hence the pressure gradient term is immediately identified, however we are yet to fully consider the deviatoric stress tensor and how it translates into the diffusion term. Note that in our full Navier Stokes Equations we utilise components of velocity in our diffusion term, hence we must find a relationship that relates stress to some form of velocity. For an incompressible fluid such a relationship is found in a unique form of the mathematical definition of a Newtonian fluid:

$$\sigma'_{ij} = \mu \left( \frac{\partial \vec{v}_i}{\partial \vec{x}_j} + \frac{\partial \vec{v}_j}{\partial \vec{x}_i} \right)$$

Note again from before that  $\vec{v}$  refers to the vector that gives the fluid flow velocity, where subscript letters indicate the components (subscript i,j,k, relate to components u,v,w of  $\vec{v}$  respectively).

It should also be noted that  $\vec{x}$  relates to the vector giving the position of an infinitesimal control volume of fluid in the field of space, again where subscript letters indicate the components subscript i,j,k relate to components x,y,z of  $\vec{x}$  respectively).

Articulated in a non-mathematical form, a Newtonian fluid is a fluid where the shear stress applied to the fluid is directly proportional to the velocity gradient (the change in velocity between adjacent fluid elements) of the fluid. [Note here that  $\mu$ , the constant of proportionality relates to the fluid's shear viscosity].

Before we substitute in elements of the matrix the reader may enquire as to why we are able to apply the mathematical definition of an incompressible Newtonian fluid to all elements of the tensor since we defined the definition as being applicable to shear stresses denoted as  $\tau$  but failed to mention relevance to normal stresses  $\sigma$ . To answer this we must consider the implications of the RHS of the definition: the term on the RHS is known as the velocity gradient, the reader can consider this term to effectively act as measure of the rate of deformation of the fluid.

Stresses are either normal stresses or shear stresses, both are able to change fluid elements: however, the former changes fluid elements by changing their volume while preserving shape whereas the latter distorts the shape of the fluid elements. We model this as two tensors: the Hydrostatic [Distortion of Volume] and the Deviatoric Tensor [Distortion of Shape] – note that in the Hydrostatic tensor the stress acting on an element is uniform in all directions leading to uniform compression in [isotropic] fluids – we are able to make such a conclusion as the trace of the tensor [the sum of all the diagonal components and hence the sum of all the normal stresses] is a non-zero value.

The deviatoric tensor considers the stresses that lead to a distortion of fluid shape, note that here the trace of the matrix [the sum of all the normal stresses] must equal 0 in an isotropic fluid:

$$\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = 0$$

This would imply that unless  $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$  that at least one of the normal stresses would need to take a non-positive value, which would lead to a distortion of shape. Hence it is appropriate to use the constitutive equation to relate normal stress to rate of distortion.

Hence, we can now begin to manipulate:  $\nabla \cdot \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$

Substituting all the elements in the tensor into an expression of change of velocity and shear viscosity, as they are all vectors, we attain the following:

$$\sigma_{xx} = \mu \left( \frac{\partial \vec{v}_x}{\partial \vec{x}_x} + \frac{\partial \vec{v}_x}{\partial \vec{x}_x} \right) = 2\mu \frac{\partial u}{\partial x}$$

$$\sigma_{yy} = \mu \left( \frac{\partial \vec{v}_y}{\partial \vec{x}_y} + \frac{\partial \vec{v}_y}{\partial \vec{x}_y} \right) = 2\mu \frac{\partial v}{\partial y}$$

$$\sigma_{zz} = \mu \left( \frac{\partial \vec{v}_z}{\partial \vec{x}_z} + \frac{\partial \vec{v}_z}{\partial \vec{x}_z} \right) = 2\mu \frac{\partial w}{\partial z}$$

$$\tau_{xy} = \mu \left( \frac{\partial \vec{v}_x}{\partial \vec{x}_y} + \frac{\partial \vec{v}_y}{\partial \vec{x}_x} \right) = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \tau_{yx} = \mu \left( \frac{\partial \vec{v}_y}{\partial \vec{x}_x} + \frac{\partial \vec{v}_x}{\partial \vec{x}_y} \right) = \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

$$\tau_{xz} = \mu \left( \frac{\partial \vec{v}_x}{\partial \vec{x}_z} + \frac{\partial \vec{v}_z}{\partial \vec{x}_x} \right) = \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = \tau_{zx} = \mu \left( \frac{\partial \vec{v}_z}{\partial \vec{x}_x} + \frac{\partial \vec{v}_x}{\partial \vec{x}_z} \right) = \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)$$

$$\tau_{yz} = \mu \left( \frac{\partial \vec{v}_y}{\partial \vec{x}_z} + \frac{\partial \vec{v}_z}{\partial \vec{x}_y} \right) = \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) = \tau_{zy} = \mu \left( \frac{\partial \vec{v}_z}{\partial \vec{x}_y} + \frac{\partial \vec{v}_y}{\partial \vec{x}_z} \right) = \mu \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)$$

$$\therefore \nabla \cdot \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} = \nabla \cdot \begin{pmatrix} 2\mu \frac{\partial u}{\partial x} & \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & 2\mu \frac{\partial v}{\partial y} & \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \mu \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & 2\mu \frac{\partial w}{\partial z} \end{pmatrix}$$



Recall our mathematical definition for nabla notation:

$$\begin{aligned}
 &= \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} 2\mu \frac{\partial u}{\partial x} & \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & 2\mu \frac{\partial v}{\partial y} & \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \mu \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & 2\mu \frac{\partial w}{\partial z} \end{pmatrix} \\
 &= \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} 2\mu \frac{\partial u}{\partial x} & \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \mu \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \mu \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & 2\mu \frac{\partial v}{\partial y} & \mu \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \mu \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \mu \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & 2\mu \frac{\partial w}{\partial z} \end{pmatrix}
 \end{aligned}$$

Finally, we substitute out the scale factor of  $\mu$ :

$$= \mu \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} 2 \frac{\partial u}{\partial x} & \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & 2 \frac{\partial v}{\partial y} & \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & 2 \frac{\partial w}{\partial z} \end{pmatrix}$$

Hence, we now calculate the components:

*X-Component*

$$\begin{aligned}
 \mu \left[ \frac{\partial}{\partial x} \left( 2 \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \right] &= \mu \left[ \left( 2 \cdot \frac{\partial^2 u}{\partial x^2} \right) + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 w}{\partial z \partial x} + \frac{\partial^2 u}{\partial z^2} \right] \\
 &= \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 w}{\partial z \partial x} \right) \right] \\
 &= \mu [\nabla^2 \cdot u] + \mu \left[ \frac{\partial}{\partial x} (\nabla \cdot \vec{v}) \right]
 \end{aligned}$$

*Recall that  $\nabla \cdot \vec{v}$  is equal to 0 for an incompressible fluid, hence*

$$= \mu [\nabla^2 \cdot u]$$

*Y-Component*

$$\begin{aligned}
 \mu \left[ \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( 2 \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \right] &= \mu \left[ \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} + \left( 2 \cdot \frac{\partial^2 v}{\partial y^2} \right) + \frac{\partial^2 w}{\partial z \partial y} + \frac{\partial^2 v}{\partial z^2} \right] \\
 &= \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} + \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z \partial y} \right) \right] \\
 &= \mu [\nabla^2 \cdot v] + \mu \left[ \frac{\partial}{\partial y} (\nabla \cdot \vec{v}) \right]
 \end{aligned}$$

*Recall that  $\nabla \cdot \vec{v}$  is equal to 0 for an incompressible fluid, hence*

$$= \mu [\nabla^2 \cdot v]$$

*Z-Component*

$$\begin{aligned}\mu \left[ \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) + \frac{\partial}{\partial z} \left( 2 \frac{\partial w}{\partial z} \right) \right] &= \mu \left[ \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial y^2} + \left( 2 \cdot \frac{\partial^2 w}{\partial z^2} \right) \right] \\ &= \mu \left[ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + \left( \frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial z^2} \right) \right] \\ &= \mu [\nabla^2 \cdot w] + \mu \left[ \frac{\partial}{\partial z} (\nabla \cdot \vec{v}) \right]\end{aligned}$$

*Recall that  $\nabla \cdot \vec{v}$  is equal to 0 for an incompressible fluid, hence*

$$= \mu [\nabla^2 \cdot w]$$

Hence our overall diffusion term for an incompressible fluid is:

$$\mu [\nabla^2 \cdot \vec{v}] \blacksquare$$

Yet the basis of the diffusion term in the compressible form of the Navier Stokes Equation remains to be answered....

The greatest problem when attempting to explain compressible viscous force is that unlike the incompressible viscous force, the density of the fluid is not uniform.

Recall that the diffusion term [and pressure gradient term] in the Navier Stokes equation is defined by the divergence of the fluid stress tensor  $\sigma$ , therefore we must define a stress tensor for a compressible fluid if we wish to understand the basis of the compressible diffusion term.

However, since this is significantly more complex than our tensor manipulation before, it is best to begin at an elementary stage:

We begin by recalling the Cauchy Momentum Equation from earlier:

$$\begin{aligned}\rho \frac{D\vec{v}}{Dt} &= \nabla \cdot \sigma + f \quad f \text{ here relates to external forces such as weight} \\ \therefore \sigma_{ij} &= -P\delta_{ij} + \sigma'_{ij} \\ \Rightarrow \rho \frac{D\vec{v}}{Dt} &= -\nabla P + \nabla \cdot \sigma'_{ij} + f\end{aligned}$$

Hence the challenge arises in the  $\nabla \cdot \sigma'_{ij}$  term, specifically the tensor  $\sigma'_{ij}$ . To be able to overcome such a challenge we need to consider the deviatoric stress tensor to a greater extent, consider how the following is true for a Newtonian fluid [note that we used a specific form of this definition for an incompressible fluid]:

$$\sigma'_{ij} = \mu \left( \frac{d\vec{v}_i}{d\vec{x}_j} \right)$$

The deviatoric [shear] stress tensor is directly proportional to the velocity gradient of the fluid. The velocity gradient term however can be divided into two terms:

$$\left( \frac{d\vec{v}_i}{d\vec{x}_j} \right) = (\nabla \cdot \vec{v} + (\nabla \cdot \vec{v})^T) - \frac{2}{3} (\nabla \cdot \vec{v}) I$$

[www.thevisualroom.com, n.d]

To avoid digressing into mathematics and physics that would inconvenience the reader due to their complexity (this is not necessarily a case that they are too complex to be explained but rather a case that attempting to explain these concepts in a manner that is intuitive and explains the notation used at each stage would lengthen this paper considerably and possibly put the quality of such a section in jeopardy). Identifying the problems associated with a stress tensor is therefore the most effective way to allow the reader to appreciate the level of difficulty involved in the compressible diffusion term:

- Numerous viscosity coefficients must be included: dynamic, second and bulk.
- Density is no longer a uniform quantity/constant, density must be considered as a function of temperature and pressure. A new continuity equation for the conservation of mass is needed.

Therefore, we will quote the velocity gradient term above and continue...

Hence, we are able to apply to substitute back in:

$$\sigma'_{ij} = \mu \left( \frac{d\vec{v}_i}{dx_j} \right) = \mu \left[ (\nabla \cdot \vec{v} + (\nabla \cdot \vec{v})^T) - \frac{2}{3} (\nabla \cdot \vec{v}) I \right]$$

Applying  $\nabla$  to the stress tensor found in  $\nabla \cdot \sigma'$  we obtain:

$$\begin{aligned} \nabla \cdot \sigma' &= \nabla \cdot \mu \left[ (\nabla \cdot \vec{v} + (\nabla \cdot \vec{v})^T) - \frac{2}{3} (\nabla \cdot \vec{v}) I \right] \\ &\Rightarrow \mu \nabla \cdot (\nabla \cdot \vec{v} + (\nabla \cdot \vec{v})^T) - \frac{2}{3} \mu \nabla \cdot (\nabla \cdot \vec{v}) I \\ &\Rightarrow \mu \nabla^2 \vec{v} + \mu \nabla (\nabla \cdot \vec{v}) - \frac{2}{3} \mu \nabla (\nabla \cdot \vec{v}) \end{aligned}$$

Hence our Navier Stokes Equation for a compressible fluid is:

$$\rho \frac{D\vec{v}}{Dt} = -\nabla P + \rho g + \mu \nabla^2 \vec{v} + \mu \nabla (\nabla \cdot \vec{v}) - \frac{2}{3} \mu \nabla (\nabla \cdot \vec{v}) \blacksquare$$

## Conclusion: Modelling Airflow & Solving the Navier-Stokes Equations

The reader will recall that at the beginning of the paper it was established and reiterated throughout the paper that to model airflow using the Navier-Stokes Equations would require using the compressible Navier-Stokes Equations, however this is not necessarily true.

In fact, the Navier-Stokes Equation used to model air is dependent on the context: specifically, the velocity of the air. This is a result of the compressibility of air varying with velocity:

- At velocities below 100 m s<sup>-1</sup> (0.292 Mach), air is considered incompressible
- As the velocity of air increases over 0.292 Mach, the degree of compressibility increases: however, the degree of error associated with the incompressible N-S Equations remains relatively negatable until transonic speeds are reached [around 0.8-1.0 Mach], at which point the degree of error is too great due to the compressibility effect, and the compressible N-S Equations are used [www.grc.nasa.gov, n.d]

Once the correct N-S Equations are identified, the flow of air can be modelled – however the manner in which this is done is not immediately apparent.

Modelling using the N-S Equations typically involves a 2-step process:

- Establishing conditions/constraints: such as surfaces, inlets and outlets [COMSOL, 2015]
- Solving the Navier Stokes-Equations and the appropriate continuity equation numerically, to find the velocity vector  $\vec{v}$  and pressure field from position  $\vec{x}$  and time  $t$

Note that the Navier Stokes Equations are typically solved numerically, in other words: the ‘solutions’ obtained by solving them are approximations of the actual solutions.

General solutions to the Navier Stokes Equations are not known to exist to date [at this stage the reader should be made aware of the fact that finding a general solution to the N-S equations is a topic that is mostly of interest to mathematicians rather than physicists], there are many known solutions to the Navier Stokes Equations however these exist for specific cases and conditions – in fact mathematicians have accepted that it is highly unlikely that a general solution for all possible cases even exists for the equations, as they are non-linear second-order partial differential equations.

However, it would be wrong to simply state that mathematics has abandoned the Navier Stokes Equations and its solutions: instead, mathematics has established the objective of proving or disproving that a solution always exists for all conditions – these solutions must have derivatives that are continuous and be defined on  $(-\infty, \infty)$ , in other words the solutions must be smooth and globally defined respectively [Fefferman, 2022].

This problem is known as the Navier Stokes Existence and Smoothness Problem, and is currently unsolved – a solution to it would not only award its solver \$1,000,000 but award physics by advancing knowledge of the characterisation of fluid flows significantly....

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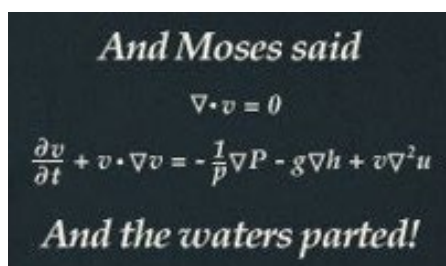
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In 2009 the then American President, Barack Obama, visited MIT in the USA where he praised the Institute's commitment to energy research and issued a strong call for the nation to lead the world in the development of new, efficient and clean energy technologies.

While there, he was gifted a T-shirt with a very special “joke” on the front. It read....



Now you can figure it out!

## NOKU JAJI

# THE SCIENCE BEHIND MAGNETIC LEVITATION TRAINS

The story goes that the idea for the magnetic levitation train was conceived when the inventor of the technology, James Powell, was stuck in a traffic jam and wondered if there was a faster method of land travel than cars and trains <sup>[1]</sup>. The mode of transportation that resulted from this idea was the futuristic concept that is the maglev train: the basic concept of the train is to reduce friction between the track and wheels by using magnetic forces to suspend the train above the track. Since the train is no longer in direct contact with the track, or guideway as it is sometimes called, the only resistance force is drag so the train can have a higher acceleration and maximum speed which means that journeys would take a lot less time. The idea of using magnets to suspend trains above a track does however predate Powell's 1968 patent such as Robert Goddard who had thought of using magnets in a very similar way 60 years prior <sup>[2]</sup>. The difference between the previous ideas and Powell's was that Powell was the first to propose they use superconductors to achieve the levitation.

### Electromagnets on the train

The electromagnets on the train are powered by the train's battery and are made of superconducting materials. Superconductors are materials that have been cooled below a critical temperature that exhibit two special properties <sup>[3]</sup>. The first property of a superconductor is that the material has no electrical resistance so if a current is passed through the material there will be no energy dissipation. The second property of the material is that weak electromagnetic fields will not penetrate the surface of the superconductor. These two properties have been essential in making the technology of maglev trains possible; the former property allows for almost no energy to be lost in the electromagnets on the train; the latter property is important as when a superconductor is placed in a magnetic field the object will be stationary in a stable state. Since superconductors only have these properties at near absolute zero temperatures, a range of 0K to 10K <sup>[4]</sup>, the trains carry liquid helium as a coolant<sup>[5]</sup>.

### Magnets in the guideway

The electromagnets of the train interact with the coils and electromagnets in the guideway to achieve three functions: levitation, guidance, and propulsion. The levitation and guidance magnets in the guideway are passive, they are usually either permanent magnets or metal coils. If a permanent magnet is used it will be put into the guideway with a specific polarity. In the case of using metallic coils the idea is that as the train passes the superconductor electromagnets induce a current into these coils; the induced current makes the coil an induced electromagnet and works in the same way as if they were permanent magnets. The propulsion electromagnets however are active as the polarity of these electromagnets need to be switched repeatedly to achieve propulsion.

### Levitation

As the name suggests this is the main aspect of a maglev train. There are two main types of methods to levitate the train: electromagnetic suspension (EMS) and electrodynamic suspension (EDS). The difference between these methods is that EMS uses the attraction of magnets while EDS uses the repulsion of magnets.

Electromagnetic suspension works by having magnets on the underside of the track and having those magnets attract the magnets above arms that are below the train as illustrated in figure 1. As shown in figure 1 the stator is attracted to the support magnet and this attraction is the upward force that causes the train to levitate. The stator is simply a magnet attached to the bottom of the guideway.

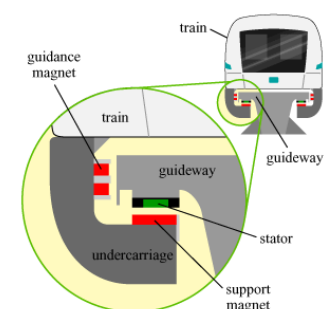


Figure 1: Diagram of EMS <sup>[6]</sup>



This method of levitation is used in many places in the world but a major issue to EMS is that the train and guideway begin to vibrate. In this configuration the train will hover at about 8-12mm above the guideway but because of this small clearance a non-linear vibration is caused that is very difficult to deal with.

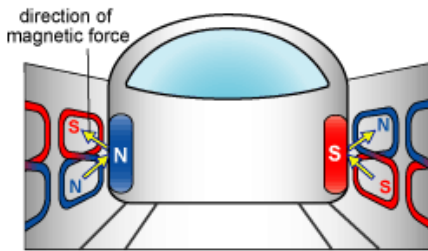


Figure 2: Diagram of EDS

Electrodynamic suspension uses the opposite mechanism by instead having the electromagnets on the sides of the train repel the magnets on the walls of the guideway. Unlike a T-shaped guideway, EDS requires a U-shaped guideway. The reason for these shapes will be further discussed when talking about guidance and stability of the train. One side of the train is the south pole, and the other is the north pole: these poles interact with the magnets in the wall of the guideway by repelling and

attracting to create an upward force, as shown in figure 2.

## Guidance

The second aspect to consider when it comes to maglev trains is the guidance and stability of the train. I will explain the way the train is stabilised using an EDS configuration, but the concept works very similarly for EMS. If there is a downward force on the train the train will approach the magnets of like poles at the bottom of the guideway, shown in figure 2. Since the distance between like poles is decreasing the magnetic repulsion force between the magnets increases so counteracts the downward force and returns to equilibrium. If there instead an upward force the distance between like poles will increase so the repulsion force will decrease. Since the repulsion has decreased the weight of the train will bring it back down to equilibrium.

The stability of a maglev train is not limited to vertical forces but also corrects for horizontal forces such as cross winds. The concept is the same as if the train is pushed towards the right wall the distance between like poles decreases so the force of repulsion between the magnet increases and counteracts the horizontal force. The stability of the system results in maglev trains, both EMS and EDS, being safer in terms of derailment than a conventional train.

## Propulsion

On a regular train the forward force is provided by the wheels contacting the track but since maglev trains are not in contact with the guideway the propulsion force is provided by the attraction and repulsion between the electromagnets in the guideway and train.

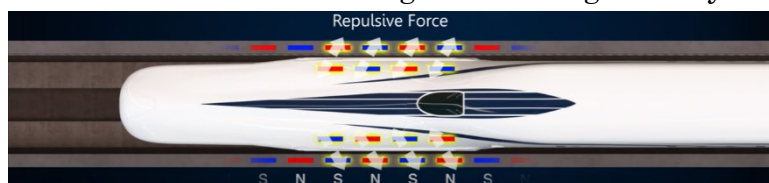


Figure 3.1: Repulsive forces being used for propulsion [7]

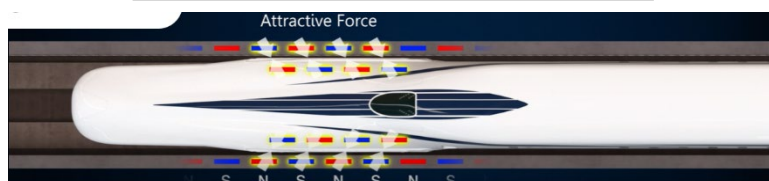


Figure 3.1: Attraction forces being used for propulsion [7]

As the train passes electromagnets of the same polarity the repulsive force acts as a forward propulsion force as shown in figure 3.1 and simultaneously the train is approaching electromagnets of differing polarities, so the attraction force also acts as a propulsion force for the train. As mentioned earlier the propulsion magnets must change polarities to ensure that the attraction and repulsion forces are acting with the motion of the train and not against it. The frequency at which the propulsion magnets alternate poles is how the speed of the train is regulated and controlled.

### **Benefits and Detriments**

The most obvious benefit to having maglev trains is how much faster they are than conventional high-speed rail. The LO Series Maglev in Japan has a maximum operating speed of 310mph and holds the land speed record for rail vehicles while the TGVPOS, which held the record before the LO Series, usually operates at roughly 200mph <sup>[8]</sup>. This massive difference in speed means that passengers and goods can travel quicker which means more money can be made as more trips can be achieved. Another benefit is comfort for both people on and around the train. Within the train passengers will feel a lot less movement as all shaking that would be felt on a normal train is removed by the increased stability of the magnetic guidance. Outside, people will benefit from how much quieter maglev trains are: a high-speed train going at 220mph will produce around 98 dB <sup>[9]</sup> as opposed to a maglev train which produces around 20 dB <sup>[10]</sup>. This drastic difference in noise production means that more maglev trains can be built closer to residential areas which would be more convenient for people who want to use them regularly.

The major detriment of maglev trains is the initial cost of building. Since the only magnets that need to be powered are the propulsion magnets and the superconductor magnets on the train, it is cheaper to operate a maglev train than a conventional high-speed train <sup>[11]</sup>. The major cost is in the construction of the track. The track of a maglev train is a lot more complex than that of a conventional train which leads to the price tag being a lot higher. Another factor that makes this worse is that a maglev train cannot be placed on pre-existing tracks as they are incompatible so to build a maglev train it is required to construct an entire new route from scratch. A calculation from the U.S Federal Railroad Administration estimated the cost would be \$100 million per mile <sup>[12]</sup> while a high-speed train in Europe costs around \$40.2-\$62.7 million per mile <sup>[13]</sup>. This large difference is usually the main reason why most countries favour new conventional high-speed trains over maglev trains especially if there is already rail infrastructure.

### **Conclusion**

Magnetic levitation technology has many benefits that make it very enticing even when the costs are known, and this can be seen by how many maglev trains are currently in construction or proposed. In the 1960s the concept was a wild futuristic approach to travel but now in the 21<sup>st</sup> century we have the resources and research to make it all a reality and only time can tell how much further this concept will evolve.

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Although **MAGLEV** trains are a relatively recent development, with the first commercial trains becoming operational in Japan in 1969, the underlying technologies date back to the first decade of the 20<sup>th</sup> century. However, a crucial breakthrough came in the late 1940s when the British electrical engineer **ERIC LAITHWAITE**, a professor at Imperial College London, developed the first full-size working model of the **linear induction motor**.

The linear motor was naturally suited to use with maglev systems as well. In the early 1970s, Laithwaite discovered a new arrangement of magnets, the **magnetic river**, that allowed a single linear motor to produce both lift and forward thrust, allowing a maglev system to be built with a single set of magnets.



## EMILIO MARTINEZ

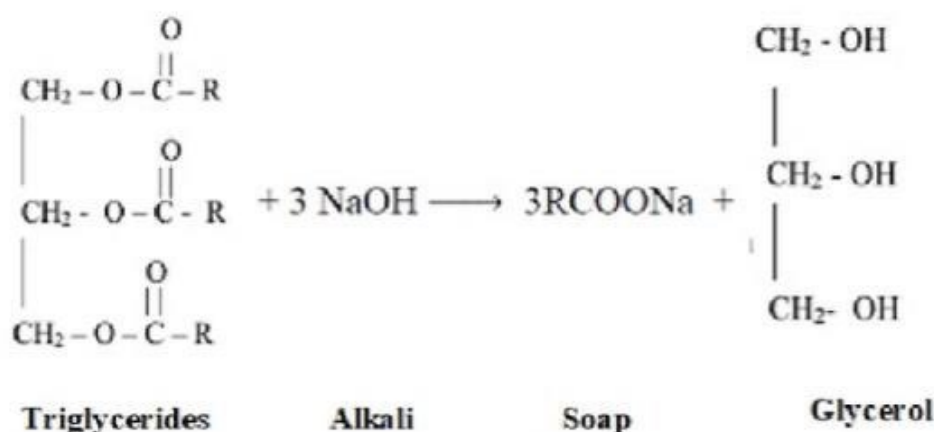
# INVESTIGATING THE SAPONIFICATION OF OIL

### Introduction:

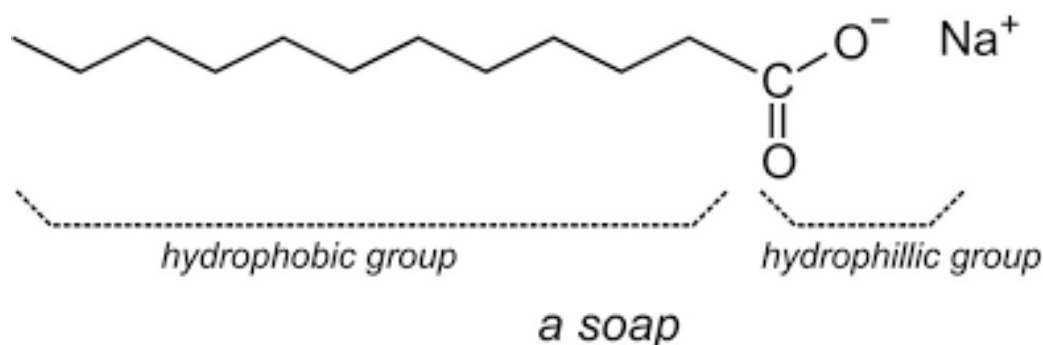
Our aim for this project was to find out the most efficient method of producing soap. To test this, we decided to heat the vegetable oil and NaOH under reflux for varying times at equal increments. Then titrate this with HCl to determine any excess NaOH that had not reacted. Later in this report we will have a graph showing rate of reaction at different times under reflux which will show the most effective length of time to heat the reaction under reflux to produce soap.

### Background research:

The basics of Saponification is an acid reacting with a base to form a salt. Saponification is a specific form of hydrolyzation, it is the process in which triglycerides (A type of fat, found in butter and oils) are combined with a strong base (commonly sodium hydroxide or potassium hydroxide depending on the type of soap desired), to form a fatty acid salt (soap) and glycerol<sup>1,2,3</sup>.



Each soap molecule contains a long hydrocarbon chain (hydrophobic) and a carboxylate (hydrophilic). The hydrophobic group in soap can bind with oil, dirt and bacteria in our hands forming a micelle whilst the hydrophilic is able to dissolve in water<sup>4,5</sup>. Hence why bacteria and dirt is washed away when using soap to wash your hands or body.



To find the efficiency of the reaction we carried out a titration of each solution. For the titration we added HCl 2M. This would first react with the  $\text{COO-NA}^+$  as it is a weak acid this forms  $\text{COOH}$ , due to this the solution turns pink due to the phenolphthalein the HCl then reacts with the strong base NaOH. The solution then returns to alkali so becomes clear again until enough HCl is added to fully neutralise the base, at which point the solution would remain pink.

## **Lab work:**

**Risk assessment** - Wear goggles, to protect eyes from concentrated HCl and NaOH.

- Careful handling hot equipment, let it cool before use.
- Appropriate closed toe shoes.
- Always keep an eye on the reflux to avoid overboiling and spillages.

## **Chemicals and Equipment** **Reflux**

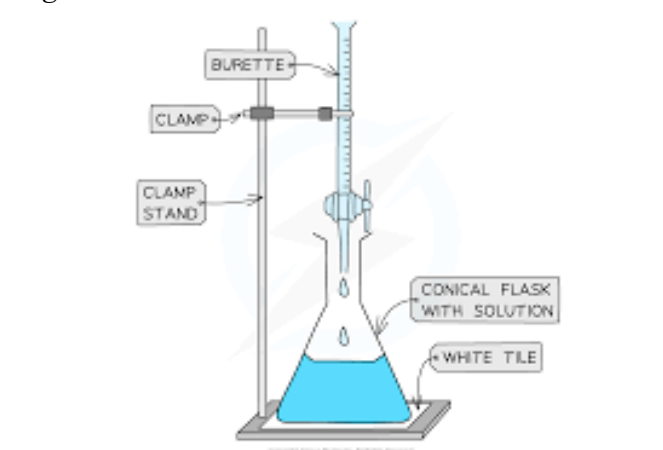
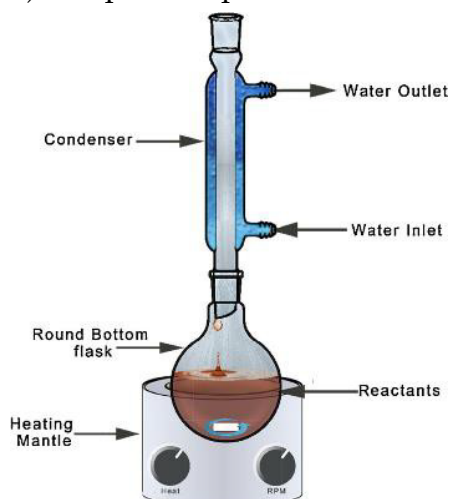
- NaOH 2M
- Vegetable oil
- Heating mantle
- Stand and clamp
- Round bottom flask
- Condenser
- Tubes x2 (access to tap)
- 25 and 10 ml measuring flask

## **Titration**

- HCL 2M
- Solution from previous reaction
- Distilled water
- Stand and clamp
- Conical flask
- Phenolphthalein
- Burette

## **Method:**

- 1) Set up equipment for reflux and titration as show in the diagram below.
- 2) Begin heating 20cm<sup>3</sup> of NaOH with 5cm<sup>3</sup> of vegetable oil in a round bottom flask under reflux at 150 Celsius using a heating mantle for 10 minutes.
- 3) Allow mixture/ equipment to cool wash all solution out of the round bottom flask with distilled water place in conical flask and add phenolphthalein
- 4) Titrate the mixture against HCl ensuring the solution stays pink before taking the titre
- 5) Use the value of the titre to calculate moles of NaOH used in the reaction.
- 6) Repeat steps 2-4 for different times in increasing increments of 10 minutes.



### **Modifications to technique:**

Our initial aim of the research project was to find the household oil after saponification produced the best soap. We would have tested this by reacting it with water and measuring the bubbles produced, alongside a titration to find how efficient the reaction was. We quickly realised that even after extensive research we couldn't find a consistent method to carry out saponification. So, we switched our target to finding the most efficient method for carrying out saponification.

#### **15<sup>th</sup> June**

Reacted 20ml of NaOH with 10ml of vegetable oil heated between 110-150°C 40 minutes observed some boiling but no change to solution. Decided to reduce volume of solution and increase time for next reaction.

#### **17<sup>th</sup> June**

Reacted 10ml of NaOH with 5ml of vegetable oil for 100 minutes. Other changes made to improve reaction were; taped down water tube to avoid spillages to improve safety and added thermometer to record more accurate temperature to try narrow down boiling point. Again observed boiling some separation at the top appeared to be soap. We attempted to separate the soap with filtration under pressure very small amount of product was successfully filtered. After this reaction as we couldn't narrow down a boiling point did some research into purity of vegetable oil and found that it wasn't pure<sup>6</sup> though eventually we decided to persevere with vegetable oil and instead increase ratio between vegetable oil and NaOH from 1:2 to 1:4.

#### **5<sup>th</sup> July**

Decided to move away from identifying the best soap to refining a method and timing to find the most efficient method for saponification hence the method detailed in the lab report. Reacted 20ml NaOH with 5ml of vegetable oil for 10 min. Observed boiling and more soap produced we then titrated it against HCl with phenolphthalein. The following lessons we followed the same method only changing the time we heated the mixture under reflux. We completed reflux reactions at 10, 20, 30, 40 and 50 minutes.

### **Results:**

Titre (cm <sup>3</sup> )	Time heated under reflux
16.4	10
15.0	20
14.2	30
13.8	40
13.5	50

Moles of NaOH	Time (seconds)	Rate of reaction (mol/sec)
0.0154	600	$2.57 \times 10^{-5}$
0.0175	1200	$1.46 \times 10^{-5}$
0.0187	1800	$1.04 \times 10^{-5}$
0.0193	2400	$8.04 \times 10^{-6}$
0.0198	3000	$6.60 \times 10^{-6}$



### Calculations:

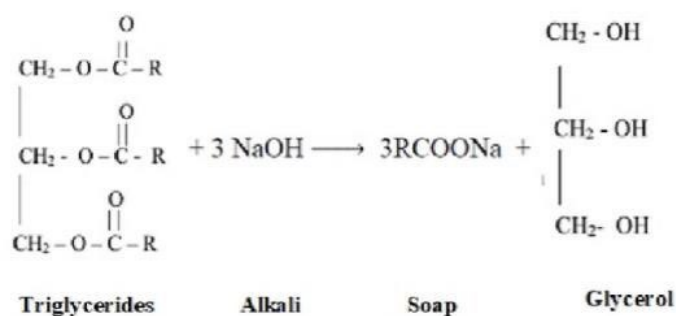
The results of the titrations give us the volume of NaOH left as excess in the reaction. In order to find the moles of NaOH used in the reaction we must find the initial moles of NaOH and then subtract the moles of NaOH left after the reaction to determine the difference.

Initial moles of NaOH = volume \* Concentration =  $20/1000 * 2 = 0.04$

Moles of NaOH left in excess =  $16.4/1000 * 2 = 0.0328$

$0.0328 * \frac{3}{4} = 0.0246$        $0.04 - 0.0225 = 0.0154$

- Must multiply by  $\frac{3}{4}$  as, moles of HCL used in titre = moles of fatty acid present + moles NaOH present.



- Using this exemplar reaction, we can see the ratio of moles of NaOH to moles of fat is 3:1.

My final calculation involved calculating the percentage uncertainty in the moles of NaOH used in the reaction.

I calculated % error in the volume of NaOH used.  $20 \text{ cm}^3 \pm 0.25 = 1.25\%$  I then calculated % error in each titre,  $16.4 \pm 0.05 = 0.3\%$

Thirdly I added the percentage uncertainties  $0.3 + 1.25 = 1.55\%$

Finally, I used the % error to give an uncertainty for each value of moles and used these in my graph.

### Conclusion:

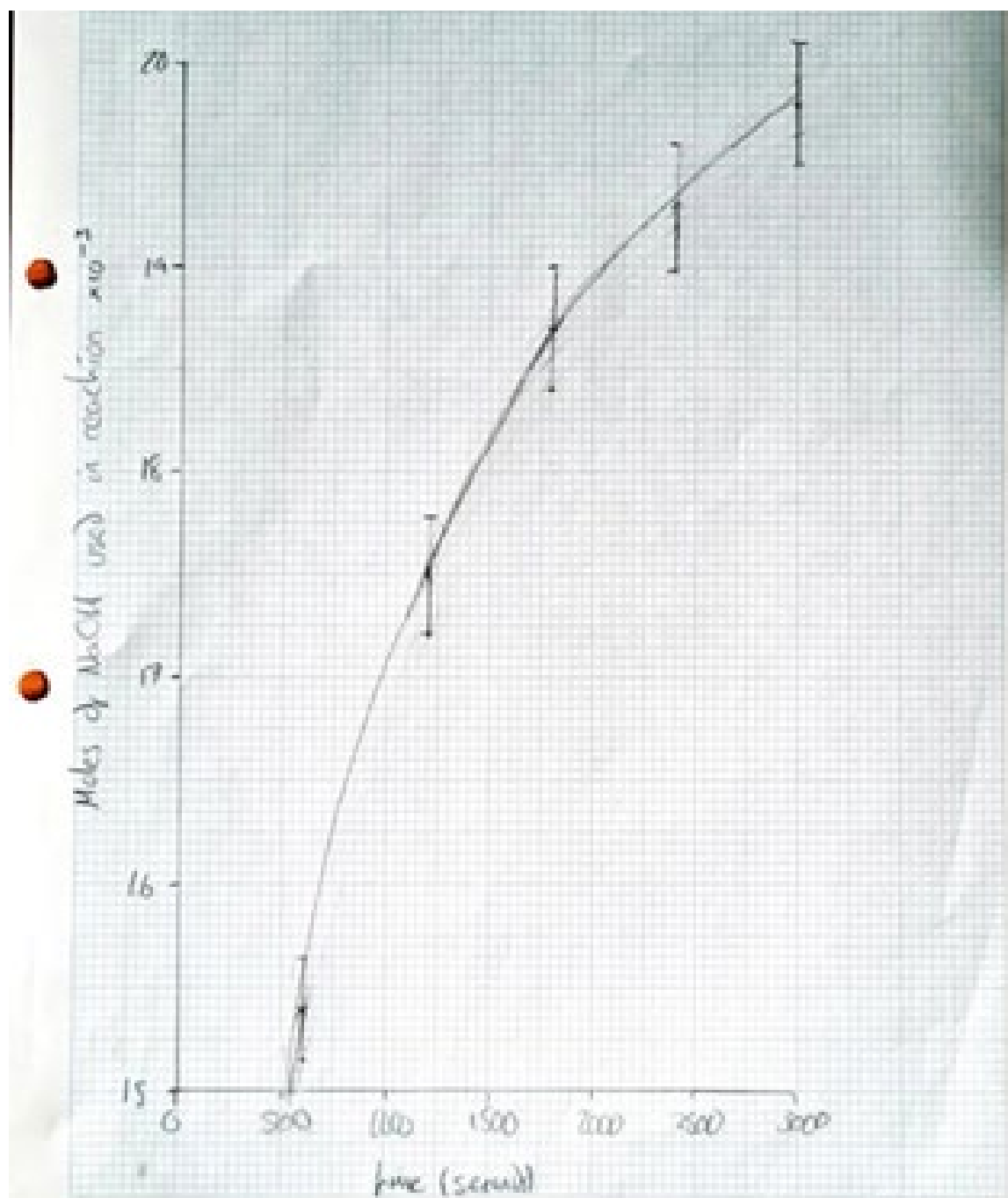
In conclusion we see the rate of reaction decreases with time as expected with a chemical reaction however based on the time taken to set up equipment and other factors we would heat the mixture under reflux for around 20 – 25 minutes for the most efficient way for the production of soaps.

Following this we would like to go forward with our original question of using different fats and oils in the production of soap. We would take physical qualities of the soap and find different ways of testing it, perhaps seeing how long a bar of each soap takes.

### Evaluation:

If I could carry out the experiment again, I would like to add different fats and oils to see how each different fat/ oil may affect the rate of reaction and if so the reason for this occurrence. A lot of time during this process felt wasted when working out a method for reflux e.g. volume of reactants and temperature of the electrical heater. Quite a few times we found the reaction would begin to over boil and finding a temperature at which the solution would evaporate without over boiling caused an issue that took up a large percentage of time. Lastly to ensure our experiment was accurate I would do repeats at each time in order to get concordant titrations.

I do however believe the route we eventually took gave us a very good introduction into saponification. It allowed us to test the efficiency of the reaction and in order to continue the investigation we would have a solid method for saponification.



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- <sup>3</sup>(Carlos M. Renoud May 6<sup>th</sup> 2017) <https://www.youtube.com/watch?v=rRTzv2co5m0>
- <sup>4</sup>(SciTech, unknown date) <https://www.scitech.org.au/experiment/the-science-of-soap/#:~:text=But%20soap%20molecules%20have%20two,wash%20all%20the%20germs%20away>
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- <sup>6</sup>(Goodman Fielder Food, 12<sup>th</sup> March 2020) <https://www.gffoodservice.com.au/idea/vegetable-oils-where-do-they-come-from/>

## THOMAS ROY

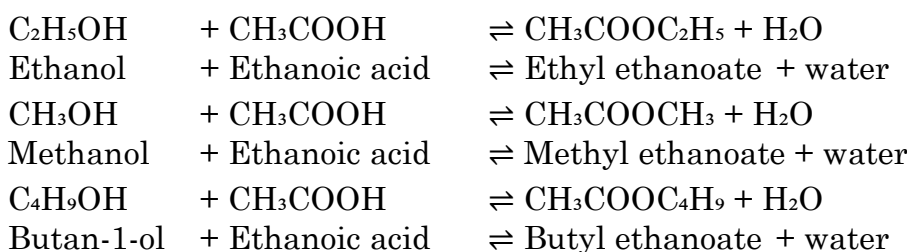
# PREPARING ESTERS AND COMPARING THEIR PHYSICAL PROPERTIES

### Introduction

When a carboxylic acid is refluxed together with an alcohol, in the presence of an acid catalyst, an ester is made, along with water. This is a reversible reaction. The ester is then separated from the water and any unreacted carboxylic acid and alcohol by using a separating funnel. The crude ester is then dried by using anhydrous sodium sulphate.

This procedure is known as Fisher esterification and samples of ethyl ethanoate, methyl ethanoate and butyl ethanoate were to be made by using this method. Their respective boiling points and densities were to then be determined. The effect of reflux time on the yield of ethyl ethanoate was to be investigated. The method was then to be modified by removing the product as it was forming, and the yield of the ester was to be calculated.

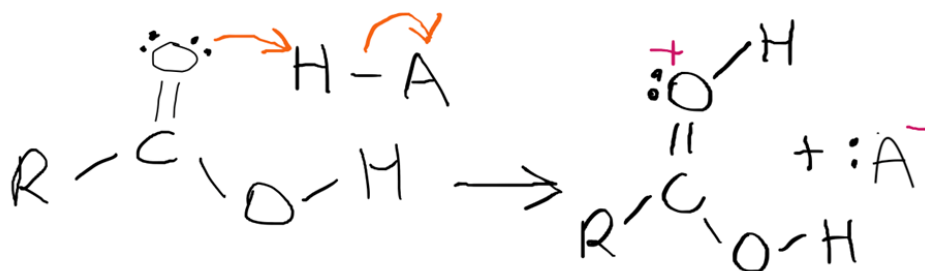
Equations for the reversible reactions:



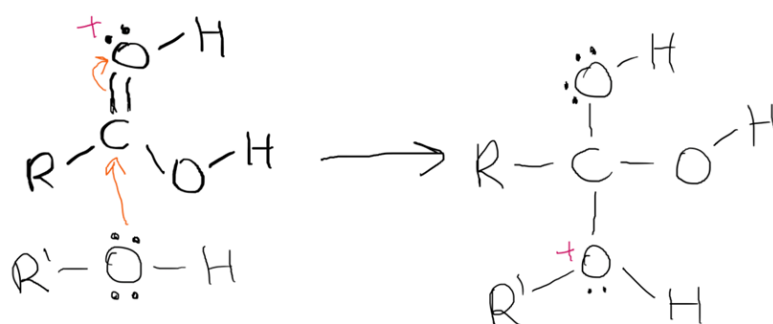
### Background research

#### Mechanism for Fischer Esterification

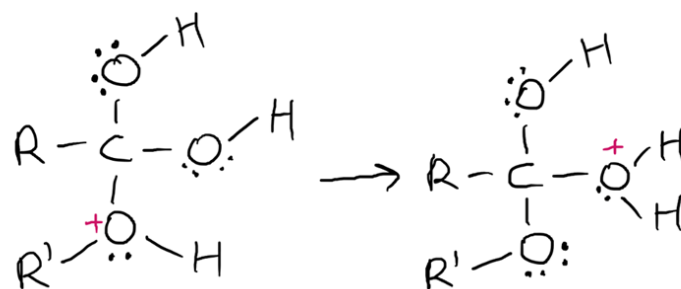
1. "The **carbonyl** oxygen is **protonated** by the acid catalyst, activating it toward a **nucleophilic** attack."



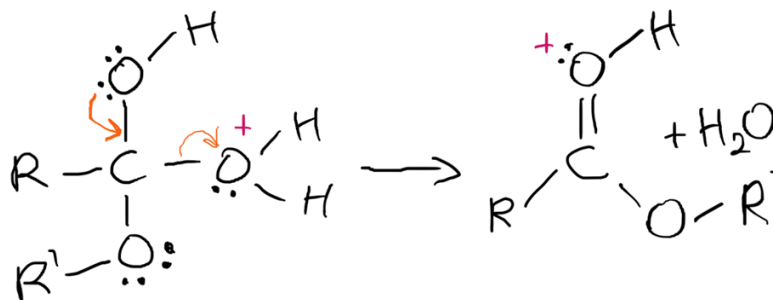
2. "The alcohol executes a **nucleophilic** attack on the carbonyl. A **lone pair** of electrons from the oxygen atom of the alcohol forms a bond with the carbonyl carbon, breaking its **pi bond** with the other oxygen. The pi bond electrons move up to the oxygen and neutralise its positive charge. This results in an **oxonium ion**."



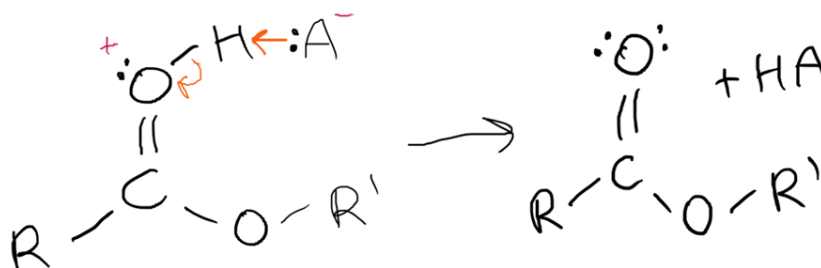
3. “A proton transfer occurs from the **oxonium ion** to the OH group”



4. “A **lone pair** of oxygen forms a **pi bond** with the carbon”, causing the elimination of water and leaving behind the **protonated** ester.



5. The “positively charged oxygen is **deprotonated**”, and so we are left with the ester. [1]



**ESTERS IN EVERYDAY LIFE!** Although our study and understanding of esters might appear to be purely for the realm of laboratory chemistry, you would be wrong. Esters play an important role in our everyday lives – in fact, animal and vegetable fats and oils are really just big esters.

The difference between a fat (like butter) and an oil (like sunflower oil) is simply in the melting points of the mixture of esters they contain. If the melting points are below room temperature, it will be a liquid – an oil. If the melting points are above room temperature, it will be a solid – a fat.

There are some terms that you may recognise associated with fats and oils: **saturated**, **unsaturated**, **monounsaturated** and **polyunsaturated**. Well, here's why – if the fat or oil is **saturated**, it means that the acid that it was derived from has no carbon-carbon double bonds in its chain. Stearic acid is a saturated acid, and so glyceryl tristearate is a saturated fat. If the acid has just one carbon-carbon double bond somewhere in the chain, it is called **mono-unsaturated**. If it has more than one carbon-carbon double bond, it is **poly-unsaturated**.



You might possibly have come across the terms **omega 6** and **omega 3** in the context of fats and oils. Linoleic acid is an omega 6 acid. It just means that the first carbon-carbon double bond starts on the sixth carbon from the CH<sub>3</sub> end. Linolenic acid is an omega 3 acid for the same reason.

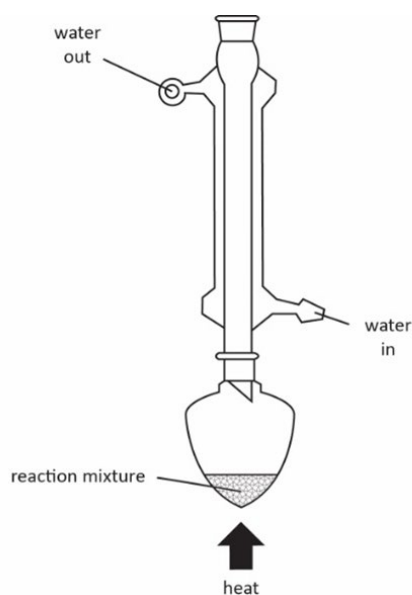
## **Procedures and observations**

### **Equipment**

- Eye protection
- Anti-bumping granules
- Pipette
- Measuring cylinders ( $50\text{ cm}^3$ )
- Measuring cylinders ( $10\text{ cm}^3$ )
- Thermometer ( $0 - 200^\circ\text{C}$ )
- Electric heater
- Retort stands, bosses, clamps and iron ring to support separating funnel
- Top-pan balance ( $\pm 0.01\text{g}$ )
- Small funnels
- $100\text{cm}^3$  glass beakers and one  $400\text{cm}^3$  glass beaker
- sample tubes with lid + glass marker pen
- wash bottle with distilled/deionised water
- conical flask ( $250\text{ cm}^3$ )
- separating funnel with stopper ( $100\text{cm}^3$ )
- Spatula
- Quickfit apparatus:
  - > round-bottomed flask ( $100\text{cm}^3$ )
  - > still head
  - > Quickfit adaptor for thermometer
  - > Liebig condenser and tubing
  - > Receiver

### **Method**

#### **Experiment 1:** Preparation of an ester using Fisher esterification



**Figure 1: Reflux setup**

1. Place a  $50\text{cm}^3$  measuring cylinder on a top-pan balance and weigh out  $0.5\text{ mol}$  of the alcohol. ( $23\text{g}$  for ethanol,  $16\text{g}$  for methanol,  $37\text{g}$  for butan-1-ol)
2. Measure out  $29\text{ml}$  of the glacial ethanoic acid into a  $50\text{cm}^3$  measuring cylinder in a fume cupboard.
3. Pour both the alcohol and the glacial ethanoic acid into the round-bottomed flask, using a funnel.

4. Cautiously add 2 cm<sup>3</sup> of the concentrated sulfuric acid dropwise, using a pipette, into the round-bottomed flask.
5. Add a few anti-bumping granules and swirl the flask.
6. Set up the electric heater and arrange the condenser above the flask in a reflux setup, as shown in Figure 1. Turn on the condenser water flow.
7. Heat the reaction mixture under reflux for a set amount of time.
8. Allow the apparatus to cool for 5 minutes and take out the round bottomed flask carefully, placing it inside the 400cm<sup>3</sup> beaker
9. Using a funnel, pour 50cm<sup>3</sup> of distilled water, and then the reaction mixture, into a separating funnel. Ensure the tap is closed.
10. Shake vigorously and remove the lower aqueous layer. Keep the upper layer of crude ester.
11. Add a further 50 cm<sup>3</sup> of water and about 25 cm<sup>3</sup> of saturated sodium hydrogen carbonate solution to the ester. Repeat the washing process.
12. Transfer the crude ester to a small conical flask and add anhydrous sodium sulphate. Swirl until the liquid appears clear.
13. Decant the dried ester into a weighed bottle and reweigh. Record the yield.
14. Use this method to prepare ethyl ethanoate using a 15 min reflux, 30 min reflux and 45 min reflux.
15. Use this method to prepare butyl ethanoate and methyl ethanoate using a 30 min reflux.

**Experiment 2:** Comparing the boiling points and densities of different esters

1. Put a 10cm<sup>3</sup> measuring cylinder on the top-pan balance and zero the top-pan balance.
2. Pour 10cm<sup>3</sup> of ethyl ethanoate into the measuring cylinder using a funnel.
3. Determine the density of the ester in g cm<sup>-3</sup> by dividing the reading on the top-pan balance by 10.
4. Repeat for methyl ethanoate and butyl ethanoate.
5. Set up the equipment into a distillation setup as shown in Figure 2., with ethyl ethanoate in the round-bottomed flask. Turn on the condenser water flow.

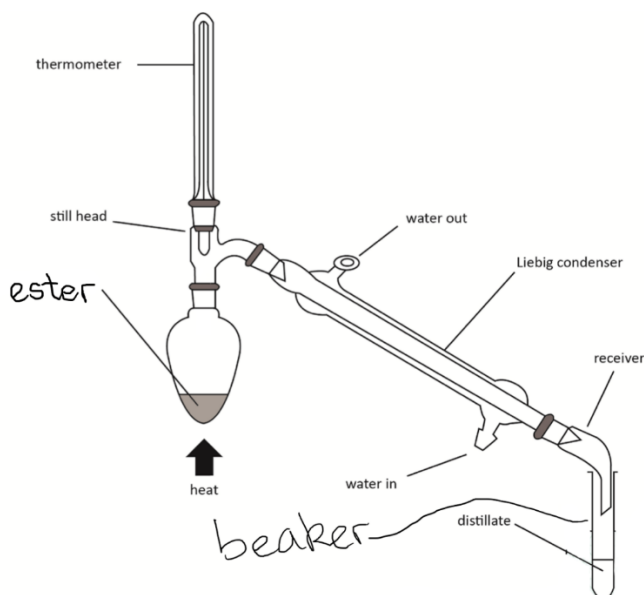


Figure 2

6. Heat the reaction mixture and note the start temperature at which drops start dripping from the receiver into the beaker.
7. Note the end temperature when there is little liquid remaining in the round-bottomed flask.
8. The start temperature to the end temperature represents the range of temperatures at which the ester boiled.
9. Repeat steps 5-7 for methyl ethanoate and butyl ethanoate

**Experiment 3:** Preparing an ester while distilling off the product

1. Perform steps 1 to 5 from Experiment 1, using methanol as the alcohol.
2. Set up the electric heater and set up the equipment into a distillation setup, as shown in Figure 3. Turn on the condenser water flow.

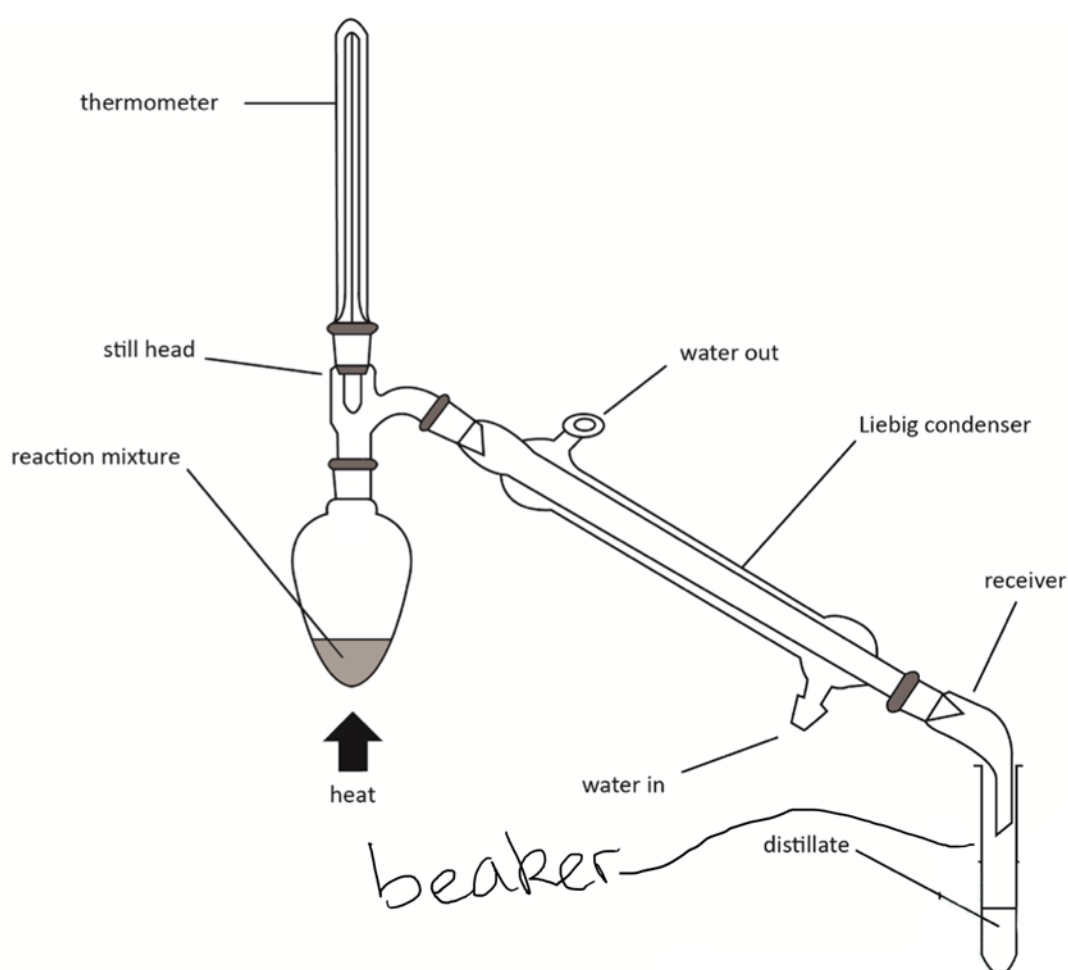














Figure 2

3. When the temperature on the thermometer starts to rise a few degrees above the boiling point of methyl ethanoate ( $57.1^{\circ}\text{C}$ ) stop collecting the distillate.
4. Perform steps 9 - 13 from Experiment 1.



**Risk assessment**

Label	Identity	Hazard information
ethanol	ethanol, $\text{C}_2\text{H}_5\text{OH}(\text{l})$	   <p>DANGER Highly flammable liquid and vapour.  <i>IDA is also:</i> Harmful if swallowed. May cause damage to organs.</p>
5.0 mol dm <sup>-3</sup> $\text{H}_2\text{SO}_4(\text{aq})$	5.0 mol dm <sup>-3</sup> aqueous sulfuric(VI) acid, $\text{H}_2\text{SO}_4(\text{aq})$	 <p>DANGER Causes severe skin burns and eye damage. <i>Wear splash-proof goggles/face shield.</i></p>
methanol	methanol, $\text{CH}_3\text{OH}(\text{l})$	   <p>DANGER Highly flammable liquid &amp; vapour. Toxic if swallowed. Toxic in contact with skin. Toxic if inhaled. Causes damage to organs if swallowed, if inhaled and through contact with the skin.</p>
butan-1-ol	butan-1-ol, $\text{C}_4\text{H}_9\text{OH}(\text{l})$	   <p>DANGER Flammable liquid and vapour. Causes skin irritation. Causes serious eye damage. May cause respiratory irritation or drowsiness or dizziness. Butan-1-ol is also: Harmful if swallowed.</p>
conc. ethanoic acid	concentrated (glacial) ethanoic acid, $\text{CH}_3\text{CO}_2\text{H}$	  <p>DANGER Causes severe skin burns and eye damage. Flammable vapour and liquid. <i>Use or dispense in a fumehood. Avoid inhaling vapour</i> <i>Wear splash-proof goggles/face shield</i></p>
saturated sodium hydrogen carbonate solution	saturated aqueous sodium hydrogen carbonate solution, $\text{NaHCO}_3(\text{aq})$	Currently not classified as hazardous at this concentration
$\text{Na}_2\text{SO}_4(\text{s})$	anhydrous sodium sulfate(VI) solid, $\text{Na}_2\text{SO}_4(\text{s})$	Currently not classified as hazardous

**Health and safety**

- Wear eye protection at all times
- Take care not to touch any apparatus that is hot
- Ensure chemicals are disposed of properly

**Results and Calculations**

	<b>Boiling Points (°C)</b>	
<b>Ester</b>	<b>Experimental value</b>	<b>Literature value</b>
<b>Methyl ethanoate</b>	47 – 65	57.1
<b>Ethyl ethanoate</b>	65 – 80	77.1
<b>Butyl ethanoate</b>	110 – 135	126.1

<b>Ester</b>	<b>Mass (g)</b>	<b>Volume (cm<sup>3</sup>)</b>
<b>Methyl ethanoate</b>	9.33	10
<b>Ethyl ethanoate</b>	9.02	10
<b>Butyl ethanoate</b>	8.83	10

Density = Mass / Volume

Methyl ethanoate density =  $9.33 / 10 = 0.933 \text{ g cm}^{-3}$

Ethyl ethanoate density =  $9.02 / 10 = 0.902 \text{ g cm}^{-3}$

Butyl ethanoate density =  $8.83 / 10 = 0.883 \text{ g cm}^{-3}$

(Actual yield/ Theoretical yield) x 100

Ethyl ethanoate theoretical yield =  $0.5 \times 88 = 44\text{g}$

Butyl ethanoate theoretical yield =  $0.5 \times 116 = 58\text{g}$

Methyl ethanoate theoretical yield =  $0.5 \times 74 = 37\text{g}$

Literature values

=  $0.932 \text{ g cm}^{-3}$

=  $0.902 \text{ g cm}^{-3}$

=  $0.8825 \text{ g cm}^{-3}$

<b>Ethyl ethanoate</b>	<b>Percentage Yield (%)</b>	<b>Actual Yield (g)</b>
15 min reflux	23.6	10.24
30 min reflux	37.5	16.51
45 min reflux	54.6	24.01

**Ethyl ethanoate**

23.6 % yield 15 min reflux

37.5% yield 30 min reflux

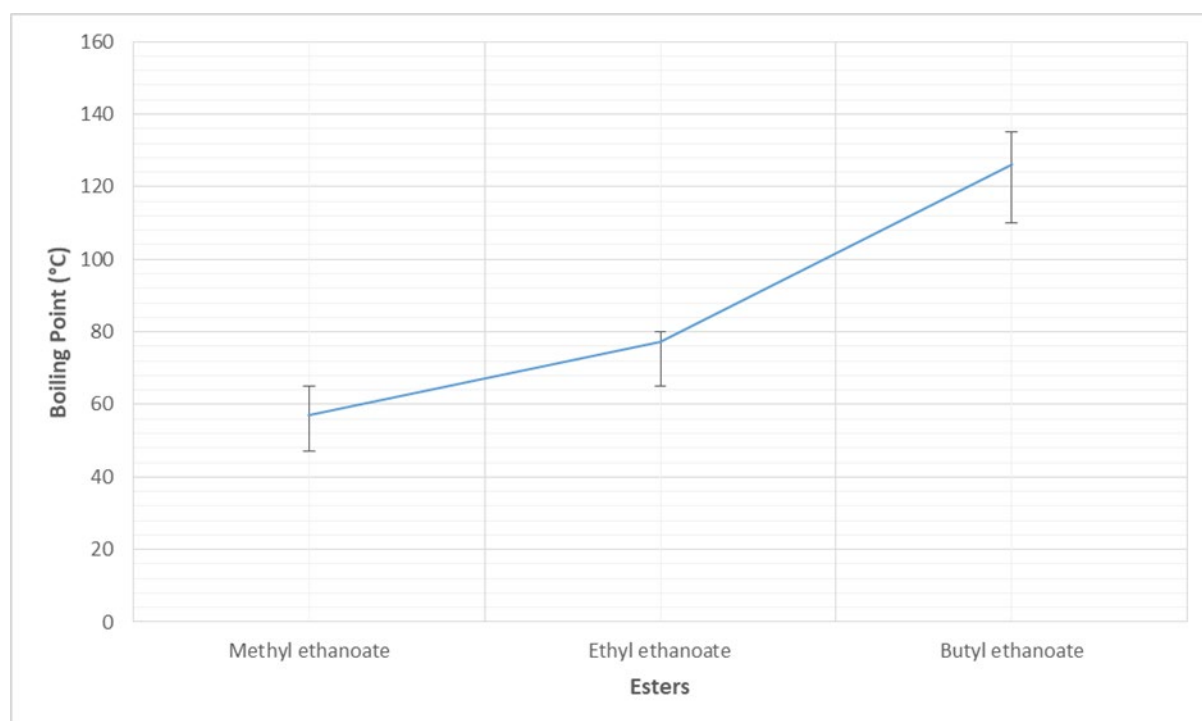
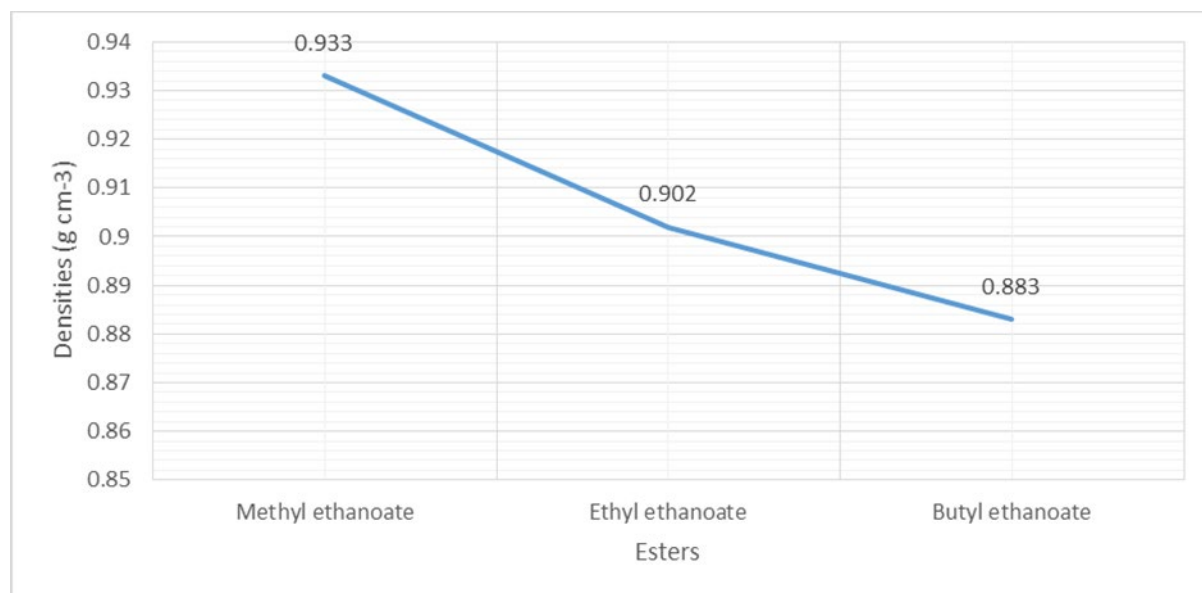
54.6% yield 45 min reflux

**Methyl ethanoate**

11.08g actual yield

29.99% yield

## Graphs



Blue line shows the literature values of the boiling points and error bars show ranges of experimental results

## Discussion

Higher reflux time means higher percentage yield which can be seen from the results as the percentage yield increases with the higher time the mixture was refluxed for.

A source of error includes yield calculated with impure ester as seen from boiling point tests. The boiling point ranges for each ester vary largely from the literature value indicating that the product made is not a pure ester. This is then included in the mass of product made increasing the percentage yield calculated. Another source of error is the error in the measuring cylinder and being unable to read precisely to the required millilitre.

Many difficulties arose such as losing products made overnight due to low boiling point esters left unsealed in warm conditions. Adding water to the products creates a suspension of water and ester where the ester is no longer able to be separated. Refluxing methanol and ethanoic acid to make methyl ethanoate might have led to the ester escaping out of the condenser since its boiling point is low.

### Suggestions to improve the experiment

Longer lab time means no products can be left overnight and reduces the number of setups of apparatus saving time. Not adding water after refluxing the mixture and adding sodium hydrogen carbonate dropwise to separate the organic layer without creating a suspension. You can check the purity of your ester by distilling the product and collecting the distillate at the boiling point of the ester.

Further investigations such as the viscosity of the different esters as well as how an excess of one of the reactants affects the yield of the product. Esters could also be prepared without Fischer esterification and by reacting an acyl chloride or acid anhydride with alcohol.

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<https://www.york.ac.uk/org/seg/salters/ChemistryArchive/ResourceSheets/esters.PDF>  
PAG5 Synthesis of an organic liquid 5.3 Oxidation of ethanol  
PAG5 Synthesis of an organic liquid 5.1 Synthesis of a haloalkane  
PAG3 Enthalpy determination 3.3 Determination of enthalpy changes of combustion

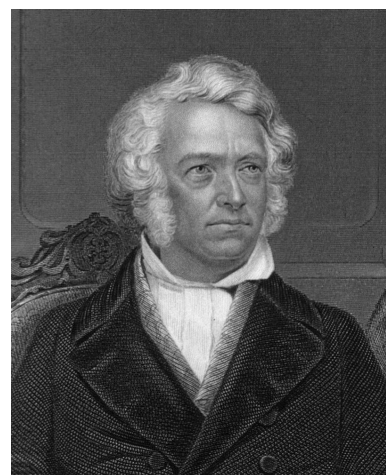
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The word **ESTER** was coined in 1848 by a German chemist **Leopold Gmelin**, probably as a contraction of the German *Essigäther*, "acetic ether".

Leopold Gmelin (2 August 1788 – 13 April 1853) was a professor at the University of Heidelberg. He wrote his *Handbook of Chemistry*, the first volume of which was published in 1817. By 1843 it had grown in the fourth edition to 9 volumes. In this edition Gmelin included atomic theory and devoted much more space to the increasingly important organic chemistry.

After Gmelin's death, an obituary was presented at the Anniversary Meeting of the Chemical Society of London and read in March 1854. The following extract gives some indication of the high regard in which Gmelin was held within the scientific community of the time:

*"The most striking feature of Gmelin's character was his thorough conscientiousness, and the purity and truthfulness of mind thence arising. He was, in the fullest sense of the word, an anima candida, —a man never satisfied with anything that may just pass for truth, but striving earnestly after that which is really true. This principle regulated his scientific inquiries, his conduct as a teacher, and, most of all, his moral and social character. In his scientific researches, he thought only of science,—not of anything external to it. Free from ostentation, and rather keeping in the background than putting himself forward, he never sought for effect or transient display, but only for solid facts and the real progress of knowledge. As a teacher, he exhibited the same absence of display and pretension, and, though not particularly eloquent, he won the confidence of his pupils by the depth and conscientiousness of his teaching, and the paternal solicitude with which he watched their progress."*



## **LEONEL JOSE**

# **SUPERCONDUCTIVITY**

### **Introduction**

Superconductors are materials that can conduct electricity with no resistance when cooled below a certain temperature, called their superconducting transition temperature. This transition temperature is different for different superconductors, ranging from a few kelvins above absolute zero to a mild spring day temperature of 15°C. As well as temperature, current and the surrounding magnetic field also affect whether a material can display superconductivity, since when the current or magnetic field is too high this phenomenon is no longer shown.

Another fascinating property that they exhibit is the ability “to displace magnetic flows completely or partly” [1], which is known as the Meissner effect, resulting in the levitation of magnetic objects. This is shown in Figure 1, where the magnetic field lines are represented by arrows and  $T_c$  stands for the superconducting transition temperature.

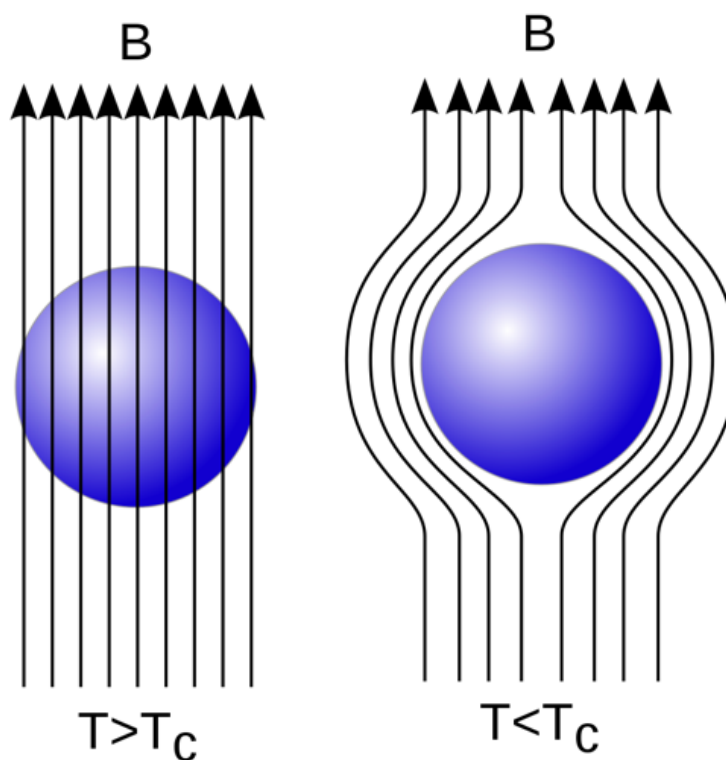


Figure 1 [8]

### **What is electrical resistance?**

Electrical resistance is defined as “a property of a component calculated by dividing the potential difference across it by the current in it” [2].

And so this gives the equation:  $R = \frac{V}{I}$

where  $R$  is the electrical resistance with units  $\Omega$ ,  $V$  is the potential difference with units Volts and  $I$  stands for current which has units Amperes.

Resistance increases when temperature increases, because the cations in a metal wire have increased internal energy and vibrate more, which leads to more frequent collisions between the delocalised electrons that act as charge carriers and the positive ions. As a result more energy is transferred to the metal cations, “so the charge carriers do more work”[2]. When a superconductor is cooled to below its superconducting transition temperature, it has no resistance, so if you have a current in a loop of the superconducting material, it will continue indefinitely with no input electromotive force, since  $V = IR$  and  $R = 0$ .

## History

H. Kamerlingh Onnes, a Dutch physicist who worked at the University of Leiden, was able to cool Helium gas to a low enough temperature that it liquified, which is at 4.2 K. He was able to measure how the resistances of several metals changed with temperature, especially when cooled to temperatures close to absolute zero. Figure 2. On the right shows the results of when he did this for mercury, and as the mercury was cooled, a sudden drop in resistance was seen, to close to 0  $\Omega$ . When this happened, he initially thought there was something wrong with the equipment, because the temperature at which the resistance was very close to 0  $\Omega$ , was the temperature at which Helium liquifies. But he did the experiment Temp. (Kelvin) and obtained similar results. Onnes did it for other metals and found that they had drop off points as well. This seemed to be a common property for the metals he observed, and he coined the term superconductivity. Onnes was awarded the Nobel prize for his work in 1913. [1] [4] [5]

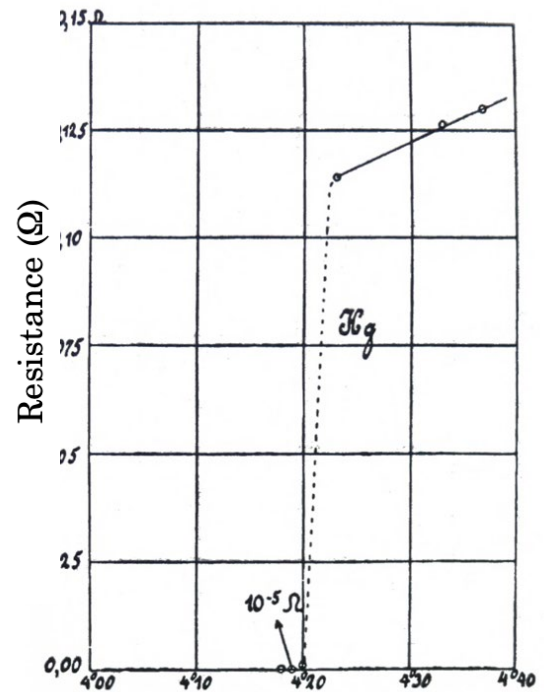


Figure 2.

## BCS Theory

This microscopic theory of superconductivity attempted to answer the “question about the underlying mechanism for superconductivity” [6]. The idea of Cooper pairs was proposed, where electrons coupled together would travel through the giant lattice of metal cations, without colliding with them, as shown in Figure 3. Usually electrons would repel each other as they have the same charge, but there is a deformation of the lattice which increases the positive charge density at that point, resulting in the attraction of electrons. At long distances, the attraction between the electrons in this region of positive charge, which results

from the displacement of the metal cations,

Is stronger than the repulsive forces between them, causing them to join together in a Cooper pair. As the electrons travel the lattice in Cooper pairs, they leave behind a “trail of positive charge” [3]. The scientists who came up with this theory were awarded the Noble prize for their work. [6]

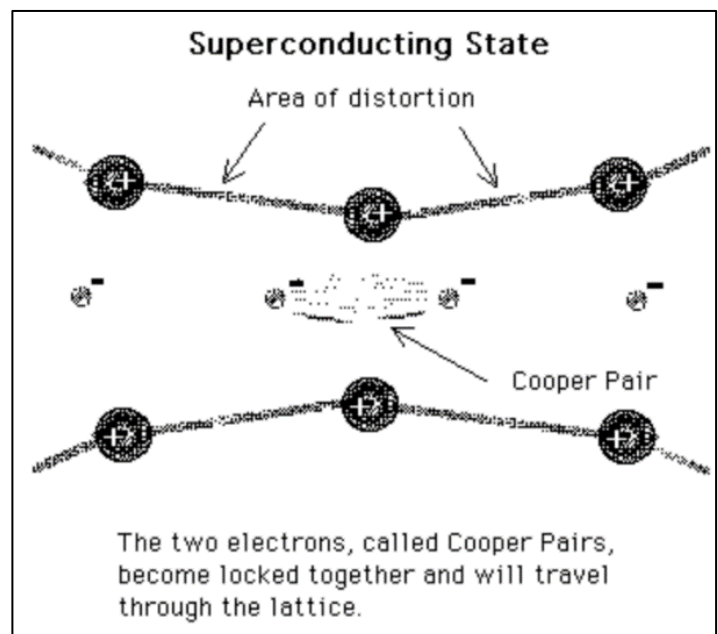


Figure 3.



## Superconducting materials

Many of the superconducting materials that were initially looked at had extremely low transition temperatures and needed liquid Helium as a coolant. Scientists strived to find more super conductors which had higher transition temperatures, with the eventual goal being to find a room-temperature conductor. Figure 4 is a graph that shows the transition temperatures of superconducting materials against the year when they were discovered.

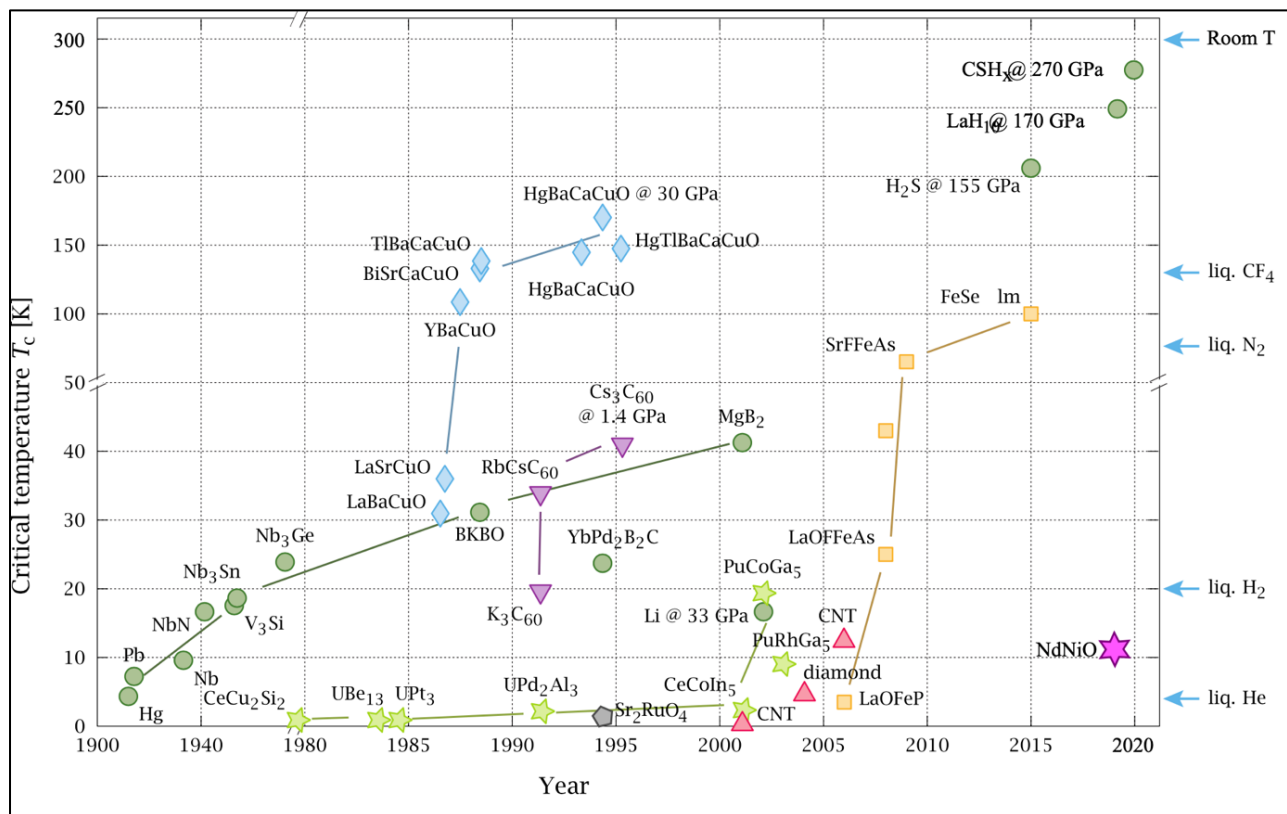


Figure 4 [9]

The super conductor with the highest transition temperature so far, was discovered in 2020, but the pressures required for the material to achieve superconductivity are “roughly 75% as extreme as those found in the Earth’s core” [7].

## Uses

Superconductors have a variety of uses, for example in maglev trains, particle accelerators and in MRI machines. These exploit the useful property of super conductors having no electrical resistance, with machines being able to use larger currents and have less power loss. [3]

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## SUPERFLUIDS!

Superconductors are turning out to be very useful indeed in a number of technologies, including magnetic levitation and MAGLEV trains which is the subject of a paper elsewhere in this Journal. They are also crucial in maintaining the powerful magnetic fields required in medical MRI scanners and the Large Hadron Collider, along with other particle colliders. Future applications may also be used in the next generation of supercomputers and ultrafast electronic switches.

But there is another phenomenon associated with matter at very low temperatures, close to 0 K, and that is **SUPERFLUIDITY**...the problem is, we don't quite know what to do with it yet!

A **SUPERFLUID** is a phase of matter capable of flowing endlessly without energy loss – they effectively have zero viscosity. This property of certain isotopes was discovered by Pyotr Leonidovich Kapitsa, John F. Allen, and Don Misener in 1937. It has been achieved at very low temperatures with at least two isotopes of helium, one isotope of rubidium, and one isotope of lithium.



Only liquids and gasses can be superfluids. For example, helium's freezing point is 1 K (Kelvin) and 25 atmospheres of pressure, the lowest of any element, but the substance begins exhibiting superfluid properties at about 2 K. The phase transition occurs when all the constituent atoms of a sample begin to occupy the same quantum state. This happens when the atoms are placed very close together and cooled down so much that their quantum wave functions begin to overlap and the atoms lose their individual identities, behaving more like a single super-atom than an agglomeration of atoms. A limiting factor on which materials can exhibit superfluidity and which cannot is that the material must be very, very cold (less than 4 K) and remain fluid at this cold temperature.

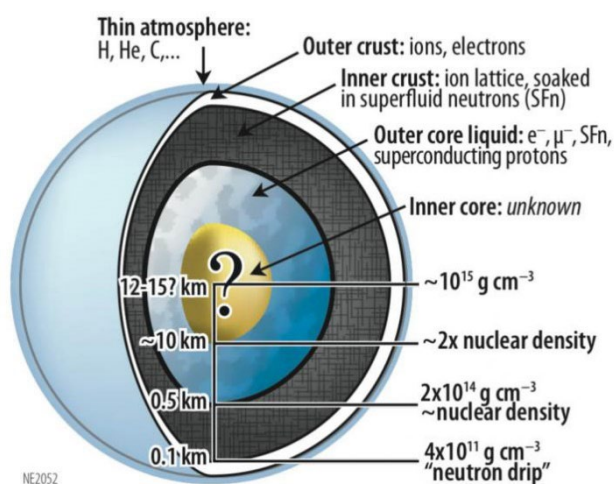
Superfluids have some impressive and unique properties that distinguish them from other forms of matter. Because they have no internal viscosity, a vortex formed within one persists forever. A superfluid has zero thermodynamic entropy and infinite thermal conductivity, meaning that no temperature differential can exist between two superfluids or two parts of the same one. They can also climb up and out of a container in a one-atom-thick layer if the container is not sealed. A conventional molecule embedded within a superfluid can move with full rotational freedom, behaving like a gas. Other interesting properties may be discovered in the future.

The potential applications of superfluids are not as exciting and wide-ranging as those of superconductors, but dilution refrigerators and spectroscopy are two areas where they have found use. Perhaps the most interesting application today is purely educational, showing how quantum effects can become macroscopic in scale under certain extreme conditions.

It is also possible that superfluids (and superconductors, too) might be present elsewhere in the Universe where we might least expect them: inside **NEUTRON STARS!** Neutron stars are extremely dense objects that form when massive stars run out of nuclear fuel and collapse in on themselves. The enormous pressure within the star forces almost all of the protons and electrons together to form neutrons. Astrophysicists would like to know more about the properties of this ultra-dense matter, and one way to do this is to study exactly how neutron stars cool.

It has been observed that the surface temperature on neutron stars drops more rapidly than expected as the star cools. This rapid cooling can be partly explained by invoking the zero-viscosity state of matter known as superfluidity. When the temperature of a neutron star falls below a certain critical value it becomes energetically favourable for neutrons inside the star to form Cooper pairs – the basic unit of the superfluid state – and that the energy released as a result could be easily removed from the star in the form of neutrinos. But it is thought that this mechanism cannot account for all of the cooling, and that superconductivity must also play a role.

The problem is that the highest temperature superconductor so far found on Earth runs at 130 K. Temperatures inside even old neutron stars exceed 1 billion K.



## **LILLI WARN**

# **THE NEUTRINO**

### **What is a neutrino?**

Like an electron, a neutrino is a subatomic particle. However, neutrinos have no charge and a far smaller mass. They are abundant throughout the universe and despite this, rarely interact with matter; making them exceedingly difficult to detect and observe.

Austrian Physicist, W. Pauli was the first to introduce the idea of neutrinos: whilst studying the energy distribution of radiation emitted by atomic nuclei, he began to question how he could explain where energy dissipated to, and so proposed the following hypothesis: "There may be a ghost-like particle with no electricity that is ejected somewhere before one knows it." This hypothesis kept his results consistent. These subatomic particles were later named neutrinos by physicist E. Fermi (neut- neutral, ion- small particle. If a neutrino was the size of 1mm, then a human would be the size of a galaxy (105,700 light years).

All matter is formed from elementary particles (atom meaning "unable to cut" in Greek). Neutrinos are one of the seventeen elementary particles discovered. When the universe was formed, neutrinos began to form inside of the sun, as well as other stars. So, when a massive star exploded, cosmic rays from the universe began to collide with the Earth's atmosphere. Furthermore, on a smaller scale, neutrinos are formed in nuclear reactors and bombs, and even bananas. Because of this, neutrinos are plentiful throughout not only earth, but the universe entirely.

Both electrons and neutrinos have properties of leptons (an elementary particle of half-integer spin that does not undergo strong interactions). Additionally, leptons include muons and tauons, as well as their antiparticles. The muon and the tau further have corresponding neutrinos: the muon-neutrinos and tau-neutrino. These are distinct, meaning that during any interactions -despite electrons and electron-neutrinos being able to be created and destroyed- the sum remains constant. This information allows physicists to divide neutrinos into three separate families: electron, muon, and tau. In which each has a corresponding charged lepton and neutrino.

### **Importance of neutrinos.**

Our early universe was a mix of high temperature particles, and as everything cooled, elementary particles changed to protons, and further to light elements (including hydrogen). As this gas gathered, stars were formed. A nuclear fission reaction occurred within the stars, producing heavier elements (such as carbon). The progression of this process over time formed the entirety of today's universe.

The main confusion of our early universe stemmed from the idea of matter and antimatter, which existed in equal quantities. When these two encountered one another, they annihilated. This would have prevented the formation of stars, and latter heavier elements would not have formed. This chain reaction would have prevented the existence of Earth altogether. It became a mystery to physicists as to how matter and antimatter did not interact despite existing in equal parts. However, the existence of neutrinos explained this unknown.

In 1998, Professor Takaaki Kajita observed muon neutrinos generated by the collision between cosmic rays and the Earth's atmosphere and noticed that the number of upward going neutrinos was only half that of downward going neutrinos. This was explained by the phenomenon of neutrino oscillations, in which a neutrino could transform to another type of neutrino whilst in flight. This event only occurs because of the neutrinos having mass, meaning that this discovery provided definitive evidence for the non-zero mass of neutrinos, and moreover the key to the asymmetry of matter. Professor Takaaki Kajita was awarded a Nobel Prize for this breakthrough.

## **Detecting neutrinos.**

Neutrinos can pass through anything, and rarely interact with matter when doing so; solar neutrinos barely collide once whilst passing through the Earth tens of billions of times. Trillions pass through our body every second, yet the detection of neutrinos did not occur until twenty years after they were named.

As neutrinos interact so little with matter the only way to detect them whatsoever involves the use of significant and extremely sensitive detectors. An average low-energy neutrino will pass through light years of normal matter before interacting with anything. Consequently, exceptionally large detectors have been built in the hope of a tiny fraction of neutrinos interacting.

So why go to all this effort for something so miniscule?

It takes approximately 100,000 years for light inside of the sun to surface. Alternatively, a neutrino can travel from the sun to the earth in eight minutes. Subsequently, studying neutrinos allows us to observe things only neutrinos can see- in this case, we can observe the inside of the sun in real time.

Neutrinos are fundamental. They travel at the speed of light and don't react with other particles. This allows astronomers to investigate cosmic microwave background radiation, which provides evidence for the expansion of the universe, initiated from the Big Bang, as a faint glow falls on Earth in every direction with a near uniform intensity. As astronomers can also use neutrinos to study solar reactions, an early warning system is provided about supernova events.

Super-Kamiokande is an example of a project that has been researching and observing neutrinos, and in doing so, has opened an inestimable number of pathways for the development of neutrino physics.

## **Super-Kamiokande.**

Super-Kamiokande is the world's largest Cherenkov detector, with initial construction in 1991, and initial observation in 1996. This research team involves collaborations from people and institutes from all around the world.

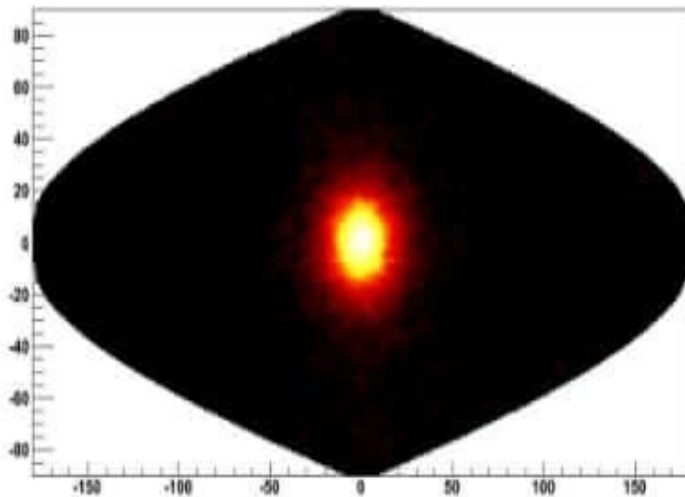
The detector consists of a stainless-steel tank, filled with fifty-thousand tonnes of water. The tank walls are covered with over thirteen-thousand photomultipliers (PMT). The detector is located a thousand metres underground, beneath the Kamioka-mine, Hida-city, Gifu, Japan.

The mountain above the detector acts as an umbrella. As primary cosmic rays pour onto the Earth, they will interact with the atmosphere, forming secondary cosmic rays (muons, electrons, neutrinos). Muons tend to lose their energy as they make contact with the surface of the Earth, in comparison to neutrinos, who rarely interact, and so do not stop. The mountain shields from cosmic ray muons, which form the background of all neutrino observations.

When a neutrino enters the detector, it interacts with the ultra-pure water, emitted Cherenkov lights. Once these are emitted, the PMT on the tank walls detect the light (how much and how long for). Using this, energy, direction, interaction point and the type of charged particle can be determined.

The Kamiokande research group were responsible for the verification of different oscillations between neutrinos and antineutrinos. Furthermore, they discovered that the composition ratio of electron-neutrinos to muon-neutrinos differed from theoretical expectation, which helped catalyse the discovery of neutrino mass.

Much of Kamiokande's research focus' on solar neutrinos, which began in 1987. The sun that is seen by neutrinos:



The sun is placed a co-ordinate (0,0).

The yellow area shows numerous events travelling from the general direction of the sun. However, a Kamiokande experiment proved that the neutrinos were flying directly from the sun.

A month into Super-Kamiokande's research into solar neutrinos, they detected neutrinos from 1987A, which is a supernova over 160,000 light years away. Developments such as these led to the formation of 'neutrino astronomy'. This involved studying the inside of stars and solar activity, including the

explosion process of stars and further the evolution of the universe's history, using neutrinos from supernova explosions throughout time.

Other than solar neutrinos, Kamiokande also studies manufactured and atmospheric neutrinos, with the discovery of a third neutrino oscillation mode due to a man-made neutrino observation in 2011. Clarifying properties of neutrinos will allow us to understand how matter was created in the early universe.

More recently, the introduction of the rare Earth element, gadolinium, has been introduced to the Super-Kamiokande detector. This has helped to improve the ability to detect and observe neutrinos and has initiated the observations of supernova relic neutrinos, antineutrino separation and proton decay. This further has helped to improve efficiency, due to the greater neutron capture cross section of gadolinium.

### **Hyper-Kamiokande and the T2K experiment.**

The successor of Super-Kamiokande will involve the use of two water tanks, with a total mass of water to succumb five times that of Super-Kamiokande. PMTs will be improved with a better resolution as well as a higher quantum efficiency. These factors will help to improve the performance of the detector, picking up even faint signals.

One tank is scheduled to start operation in 2026, the other in the following few years. Many have suggested locating the second tank elsewhere (for example, in Korea) to provide a longer baseline to sit on the second oscillation peak for the T2K beam, reducing any systematic errors. The T2K beam refers to the T2K experiment, which sent an intense beam of muon neutrinos from the East coast to the West of Japan (over 295km). The T2K experiment was the first to see an indication of neutrino appearance because of a muon neutrino beam.

### **Conclusion**

To conclude, it is clear how fundamental neutrinos are to understand our world, as they allow us to view the progression of the universe. They are the key to the existence of everything (in regards neutrino oscillations) and hold information that would be otherwise invisible. It is near to oxymoronic how much knowledge something so small and inactive can hold. Despite the difficulty of detecting them, it is undeniable that the sedentary property of neutrinos is what make them so important and put purpose into the projects involved in researching neutrino physics- as some believe that they are the key to understanding why our universe is composed of matter rather than antimatter. Harnessing the information of neutrinos will give insight to the secrets of our universe, which will not only provide an abyss of further research, but allow us to apply the information, revolutionising the world as we know it.

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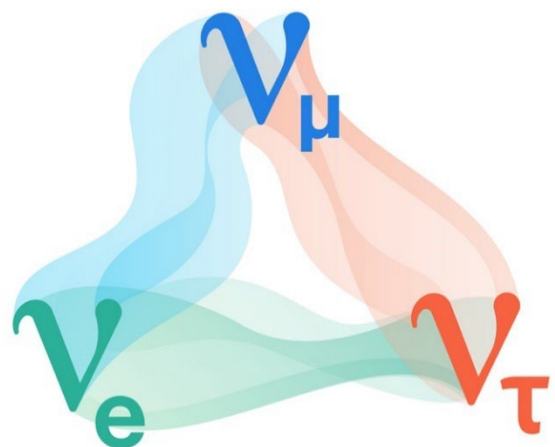
## NEUTRINO – THE “GHOST” PARTICLE

Since their theoretical beginnings in the 1930s, neutrinos were initially thought to have no mass, amazing as this may sound. After all, how can a physical particle of matter exist if it has no mass? Their existence was proven in the mid-20<sup>th</sup> century from observing radioactive decay in the laboratory. In 1965, the first neutrinos were discovered in nature, by studying cosmic radiation from the Sun. The task was set to figure out once and for all whether they had mass or not.

The problem was: exactly how can you measure the mass of the lightest particle of matter in the Universe? The answer turned out to be **neutrino oscillations**. In 1998 a Japanese physicist, Takaaki Kajita, used the Super-Kamiokande neutrino detector in Japan to discover that neutrinos produced when cosmic rays struck Earth's atmosphere were 'disappearing' as they travelled to the detector. Then, in 2002, Arthur B. McDonald, of the Sudbury Neutrino Observatory in Canada, reported that incoming electron neutrinos from the Sun were metamorphosing into muon- or tau-neutrinos. Electron-neutrino  $\nu_e$ , muon-neutrino  $\nu_\mu$  and tau-neutrino  $\nu_\tau$  are three kinds of neutrinos (named for particles they are associated with: electrons, muons and taus). What McDonald and

Kajita had together found was that neutrinos were changing from one kind to another as they travelled – a property called neutrino oscillations – which is definite proof that the particles have mass.

The Nobel Foundation decided to award Takaaki Kajita and Arthur B. McDonald with the Nobel Prize for Physics prize in 2015 for their discovery.



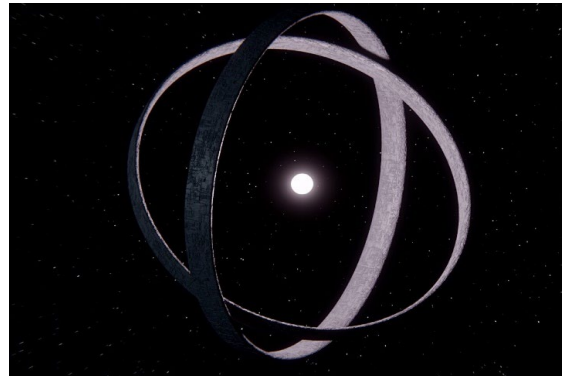


## **EMIL CHERIYAN**

# **THE DYSON SPHERE**

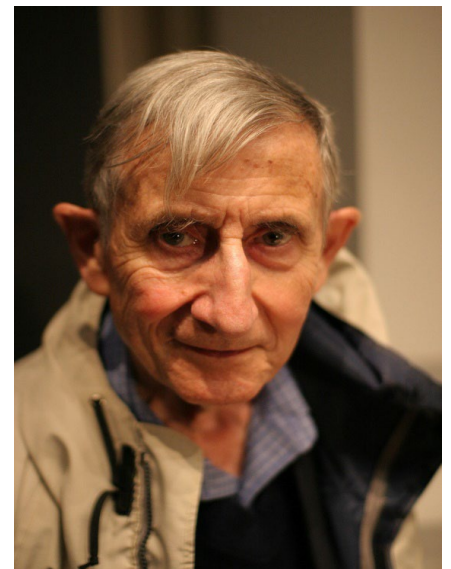
### **Dyson sphere**

The Dyson sphere is a theoretical megastructure which completely covers a star, and then captures and converts a big amount of solar energy. This structure is mainly a concept which is used to give an idea of what the future civilisation will look like in how they travel in space and expand the territory and eventually using the energy to terraform and create new technology. As the planet itself has a limited amount of resources in comparison to the regenerating amount of energy from the sun through nuclear fusion. As the current human technology is very strictly controlled by how much energy that is available to use, so if the energy provided is practically infinite then there will be leaps and bounds into the discovery of new technology that will create a better civilisation and future.



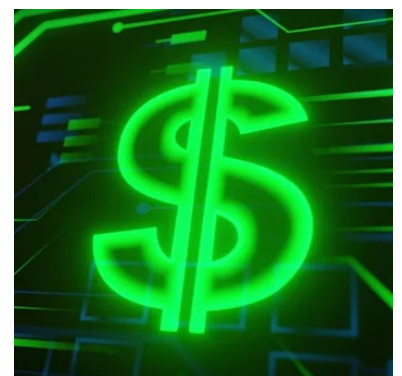
### **Origin of Concept**

The theoretical structure was introduced by Freeman Dyson, a physicist and mathematician, who along with the theory stated that there will come a time that mankind will increase their energy demand to the point that it equals the total energy output of the sun and beyond. [1] And so as a solution he put forward a series of structure that would orbit the sun or a star, which was later named a shell, with the only purpose of intercepting and collecting the energy that is produced from the star. Dyson has only focused on the issues in energy collection but not the actual structure of the Dyson sphere. It is to be noted that Dyson was first inspired by a science fiction novel 'Star Maker' by Olaf Stapledon, alongside J.D. Bernal [4], so he did not discover it but made it famous. Even to this day he hates being named after the structure as he openly credited it to [3] 'Star Maker' and wished it was named something else. [2] He was still credited on this work in his 1960 paper on the "search for Artificial Stellar Sources of Infra-Red Radiation."



### **Feasibility**

Currently the possibility of making a Dyson sphere is impossible and is far beyond humanity's engineering capacity. Merely a theoretical concept. The manpower and the resources needed to obtain, send out and maintain a single Dyson sphere will exceed present day capabilities. The cost would be completely unimaginable as just a shuttle is billions of pounds, so a structure that is millions or billions of times bigger would be an immense number. Some solutions provided where to put the Dyson sphere around smaller stars as to reduce the amount of materials needed, like a [5] white dwarf or [6] a pulsar.



## Variants

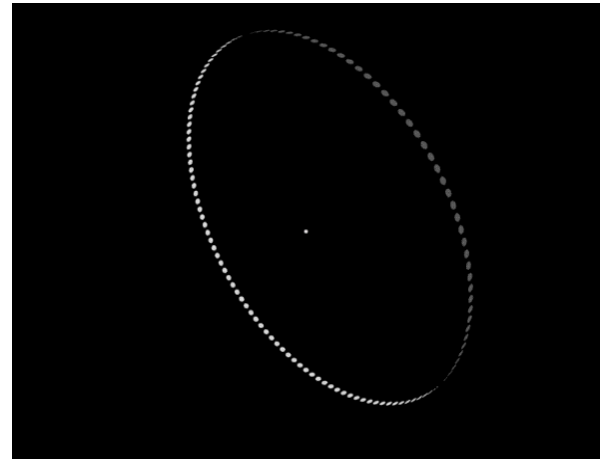
The actual structure of the Dyson sphere was never specified [7] so it is entirely possible for there to be different versions of this shape.

### Dyson swarm

This consists of a large amount of independent solar power satellites orbiting in a dense formation around the star.

### Advantages

- This is much more useful as individual components like the satellites can be controlled such as the size of it and also since it is individual the number of satellites used in the swarm can be controlled and be increased one at a time [8]
- Also, sometime in the future, it can be possible to create a type of wireless energy transfer technology to transmit the collected energy in the satellites and send it back to earth.

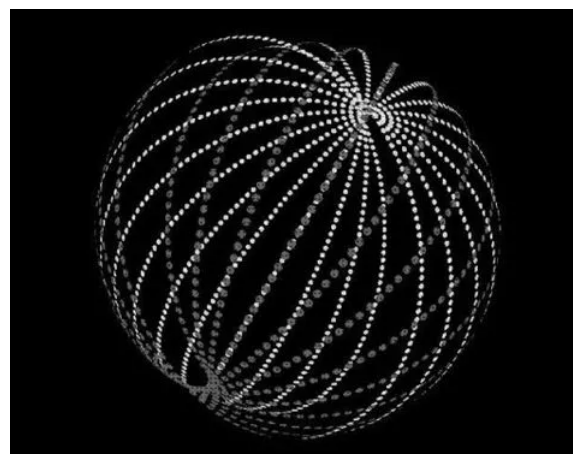


### Disadvantages:

- Orbital mechanics is extremely complex. The countless amount of arrangement that could be possible to put into around the sun can be very difficult, especially when taking in a lot of factors, like gravity, the simplest arrangement is just as ring of satellites called the Dyson ring, in which all of them compose of the same orbit, once more rings are added the complexity increases because the satellites will overlap each other reducing the absorption of the solar energy periodically [9]

## Dyson Bubble

This is similar to the Dyson ring, but it is multiple rings. These satellites are suspended by big light sails that use radiation pressure to counteract the stars pull of gravity, the satellites will not hit each other nor overlap each other. The ratio between the radiation pressure and the stars pull of gravity is constant [11], the distance between the sun and the satellites can therefore be varied. The light sails or solar sails works by capturing the energy from light particles as they bounce off a reflective surface, each light particle has momentum, and when it strikes a reflective surface, it imparts that momentum to the reflective sheet, the density of the satellite sheet or surface has to be light and so therefore currently on a graphene sheet is the lightest density with  $0.37\text{mg per m}^2$  [12], however it is not sold commercially as bulk.



## Stellar engines

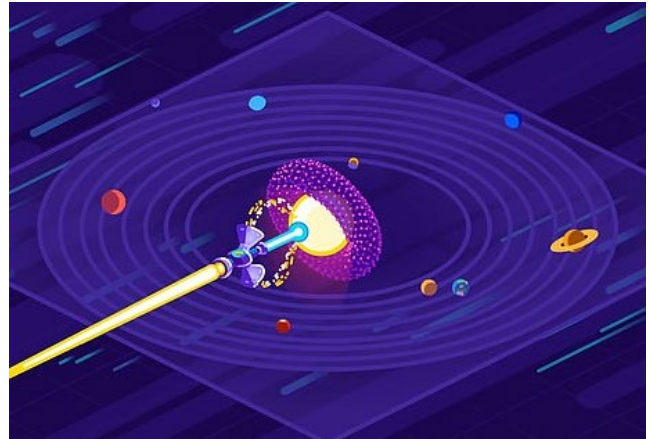
One of the many structures that can harness the energy collected for useful purposes from the Dyson sphere is a stellar engine, sounding like from a sci-fi movie. Imagine that in a few 100 years that on the news, scientists had announced that the planet was doomed to perish to a supernova near to our solar system, what could possibly save us?

Thus, in the case of a cosmic catastrophe our only hope would a stellar engine a massive machine which steer the solar system through the galaxy.



## Caplan Thruster

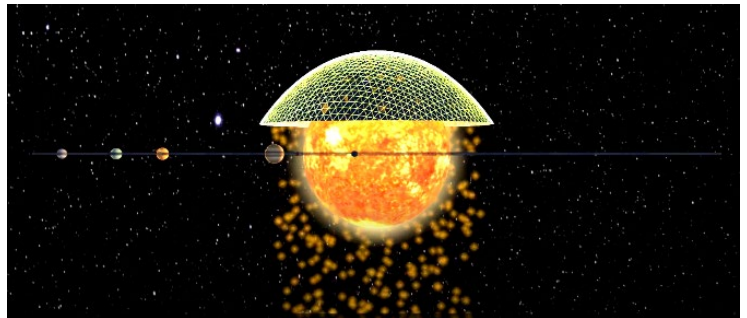
- 1) Firstly [16], a Dyson swarm or (another variant of the Dyson sphere) would have to be present in which a quintillion satellite would collect the sun's energy and reflects back, to a point like a ray of light going through a magnifying glass, to the sun, lifting billions of tons of mass of the sun in the process
- 2) Particle collector- the engine uses huge electromagnetic fields to collect the mass, separate the solar hydrogen and helium and funnel it to the accelerator and reactor.
- 3) Nuclear Fusion Reactor- the helium is burned explosively in a thermonuclear fusion reactor. A jet of radioactive oxygen produces the thrust that moves our stellar engine
- 4) Particle Accelerator- to prevent the engine from crashing into the sun, a particle accelerator shoots a jet of hydrogen back at the sun, pushing it in the direction we want to go.



## Shkadov Thruster

This uses sunlight instead of jets of matter

- 1) Parabolic mirror- By reflecting up to half the sun's light in one direction it pulls it in the other. The mirror must be very light, made of extremely thin reflecting foil.
- 2) Dangers & Limitations- to prevent burning or freezing our plants with too much or too little sunlight, the Shkadov thruster needs to be over the sun's poles, so we can only move the solar system vertically, limiting our travel options.



## Kardashev Scale

[12] "The Kardashev scale was developed as a way of measuring a civilization's technological advancement based upon how much usable energy it has at its disposal."

The Russian astrophysicist, Nikolai Kardashev designed the scale in 1964. It has 3 base classes, each with an energy threshold of energy for each level: Type I ( $10^{16}$ W), Type II ( $10^{26}$ W), and Type III ( $10^{36}$ W). This can be extended to the scale

to Type IV ( $10^{46}$ W) and Type V (this would be including energy from our own universe and multiverse if there is one). Other levels beyond this are currently unimaginable in financial and thought wise

At the moment we are on track to reaching only type I, in 100-200 years!



Once we receive the energy from the sun it can be possible to travel to other stars, galaxies to harness their energy as well.

[14] Type 1: Type 1 civilization will be able to start space travel within its solar system to nearby planets. It would also be able to colonize the moons of its home planet. Perhaps even begin terraforming some of the planets in its solar system.

[15] Type 2: A Type 2 civilization has control over their solar system and may be able to harness the power equivalent of a single star. It extracts fusion energy, information, and raw materials from multiple solar systems; it is capable of evolutionary intervention, interstellar travel, interstellar.

[16] Type 3: Type 3 Civilizations are capable of accelerating celestial bodies within their local galaxy to speeds approaching the speed of light, including planets and even entire stars.

Type 4: Type 4 civilization would be able to gather all the energy from the universe and be able to alter it in any way they see fit.

Type 5: Type 5 civilization could do the same but to the multiverse assuming that there is a multiverse.

Type 6: Type 6 civilization is God-level, they can control time and space

Type 7: This is so advanced we can't even imagine it yet. [13]

## **Terraforming**

From the energy of a Dyson sphere, it is easily possible to create a technology to terraform nearby planets or exoplanets that have a similar condition to earth. Also, the energy provided by the structure is enough to vaporise the whole solar system, so it is easily able to dismantle or destroy a planet to receive its raw materials to be used to maybe build another custom planet with preferable conditions related to modifying the atmosphere, temperature, surface topography to something relatable to earth and therefore habitable.



## **Conclusion**

Personally, I think the Dyson Sphere is a major point in the future and should definitely be looked on or used as it can kickstart mankind's leap into energy levels and use and the amount of new technology that can be possible that were only concepts as insufficient amount of energy before can now be possible. Alongside the Kardashev scale it really excites me thinking that we aren't even level 1 and makes me think what could be possible in the future, especially if there are other organisms that are at a higher level than us, could this be the reason we never end up meeting other organisms other than us? That they are simply very advanced that we can't detect them. There is also the theory that there might have been a species or organism before the age of humankind who had grown too advanced in technological levels and incurred a cataclysm inevitably causing their demise. Maybe we are in a second big bang?

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The author of the above paper includes a passage: *"It is to be noted that Dyson was first inspired by a science fiction novel 'Star Maker' by Olaf Stapledon, alongside J.D. Bernal, so he did not discover it but made it famous. Even to this day he hates being named after the structure as he openly credited it to 'Star Maker' and wished it was named something else."*

**WILLIAM OLAF STAPLEDON** (10 May 1886–6 September 1950) was a British philosopher and author of science fiction who is little known today among the younger science fiction/fantasy community. However, he was one of the most influential science fiction writers of the first half of the 20<sup>th</sup> century. So many writers who would become famous in the genre after Stapledon's death freely credit him with inspiring their own thoughts and styles.

Stapledon's writings directly influenced Arthur C. Clarke, Brian Aldiss, Stanisław Lem, Bertrand Russell, John Gloag, Naomi Mitchison, C. S. Lewis, Vernor Vinge, John Maynard Smith and indirectly influenced many others, contributing many ideas to the world of science fiction. The "supermind" composed of many individual consciousnesses forms a recurring theme in his work. *Star Maker* contains the first known description of what are now called Dyson spheres. Freeman Dyson credits the novel with giving him the idea, even stating in an interview that "Stapledon sphere" would be a more appropriate name.



*ARVIN GEORGE and NOKU JAJI*

## **EFFECTS OF CONCENTRATION ON THE ELECTROMOTIVE FORCE OF AN ELECTROCHEMICAL CELL AND ITS VIABILITY TO MEASURE THE pH OF ACIDIC SOLUTIONS**

### **Abstract**

This paper has been written to determine if there is a relationship between the concentration of a solution and the resultant electromotive force (EMF) of an electrochemical cell that uses said solution. Furthermore, the paper explores if this same relationship and similar set-up can be used as a pH probe. The first experiment conducted was to determine the relationship between concentration and EMF: this was achieved by constructing an electrochemical cell, changing the concentration of one of the half cells and recording the change in EMF shown on a multimeter. This experiment showed a clear logarithmic relationship between concentration and EMF. The second experiment conducted was to see if a similar set-up could be used as a pH probe by recording how EMF of the cell changed in solutions of different, acidic pHs. This experiment showed that pH and EMF had a linear relationship, and that the set-up was a viable method to measure the pH of acidic solutions. The specific set-up used in the experiments has the limitation of requiring the same metal electrode as the metal ion in the solution if measuring concentration and the reading for pH can have a wider range than required for an accurate measurement, but this can be mitigated with more precise instruments.

### **Introduction**

The pH of an acidic solution is the measure of how strong of an acid it is. The pH scale is a logarithmic scale that is defined as the negative log of the concentration of  $H^+$  ions.

$$pH = -\log[H^+]$$

Figure 1: Equation to calculate pH given the concentration of  $H^+$  ions in the solution

An electrochemical cell uses a redox reaction to produce a difference in charge between the two electrodes that results in a current that can be used to power electrical components. The difference in charge between the two electrodes, potential difference, is determined by the conditions in which the redox reaction takes place such as: area of contact between the electrode and electrolyte, the presence of a catalyst, temperature of the system, and concentration of the electrolyte.

An oxidation reduction reaction, or reaction, is a reaction where at least one of the reactants will lose electrons, be oxidised, and at least one of the reactants will gain an electron, be reduced. The movement of electrons and change in oxidation number is shown in half equations. Such a reaction is shown in the equations below:

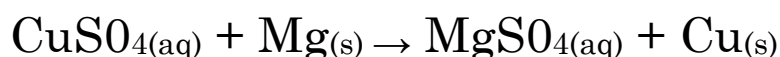


Figure 2.1: Reaction between copper sulfate and magnesium.

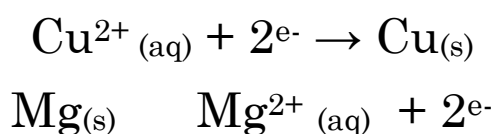


Figure 2.2: Half equations of figure 1.1



Fundamentally, electricity is the movement of electrons through a wire or component and an electrochemical cell uses the exchange of electrons in a redox reaction to drive a current. By separating the reactants and connecting them with wires to complete the circuit, the electrons are forced to travel through the wire to move between reactants. If a component is placed in this circuit, it will be powered by the redox reaction.

An important metric when it comes to the resultant EMF is the standard electrode potential of the material being used as an electrode. Standard electrode potential is a measurement of how willing a half-cell is to reduce; this is shown as the number of volts required to reduce the half-cell relative to a standard hydrogen half-cell.<sup>[2]</sup> The standard concentration of the half-cells is  $1\text{mol dm}^{-3}$  and a hydrogen half-cell is defined as  $0.00\text{V}$ . If a half-cell connected to the standard hydrogen electrode releases electrons, it is described as relatively negative so is given a negative standard electrode potential.<sup>[3]</sup> When making an electrochemical cell in standard conditions the resultant electromotive force is the difference between the standard electrode potentials of the two electrodes.

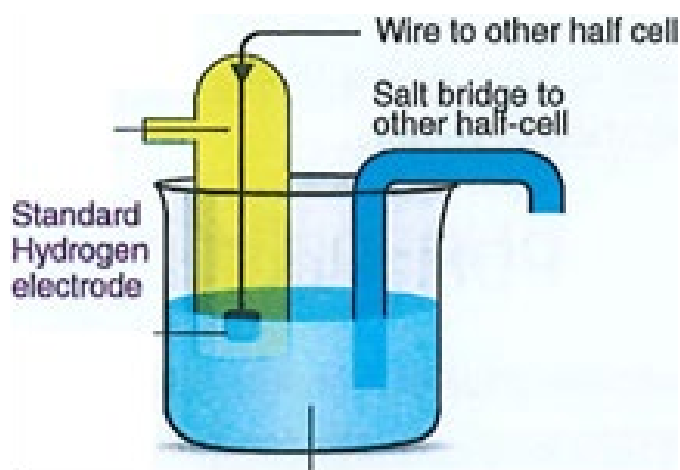


Figure 3: Diagram of a standard hydrogen electrode. <sup>[4]</sup>

### Initial trials

When starting this research, we conducted three initial trials to accomplish two goals: to obtain a design for an electrochemical cell that we could use and to show that there is a link between concentration and electromotive force.

	$\text{CuSO}_4$ ( $\text{mol dm}^{-3}$ )	EMF (V)
Preliminary Experiment 1 15/06/2022	0.4	1.977
	0.2	1.953
	0.1	1.918
	0.05	1.906
	0.025	1.900
Preliminary Experiment 2 15/06/2022	0.3	1.903
	0.2	1.949
	0.1	1.938
Preliminary Experiment 3 29/06/2022	0.5	1.844
	0.4	1.821
	0.3	1.815
	0.2	1.815
	0.1	1.811

Figure 4: Table for the three initial trials conducted

These initial experiments were not accurate as they were meant to be quick and rough. The first experiment conducted showed proof of concept for the cell design chosen- shown in figure 5- and shows a rough trend that as concentration decreases so does the EMF of the cell but it wasn't obvious to see what the trend was specifically. The second experiment had a lot less data points but there was an anomaly as the EMF peaked at 0.2 mol dm<sup>-3</sup> but as there are so few data points no conclusions can be made. The third experiment reinforced the findings of the first experiment but nothing significant was discovered from this experiment.

In the first experiment accurately conducted the concentration of copper sulfate was changed in a linear fashion but the results obtained showed no change of EMF so, in our final experiment we used serial dilution to change the concentration logarithmically.

CuSO <sub>4</sub> (mol dm <sup>-3</sup> )	EMF (V)
1.0	1.720
0.8	1.720
0.6	1.720
0.4	1.720
0.2	1.72

Figure 5: Table of the first accurate experiment

## Electrochemical Cell Design



Figure 6.1: Initial design of cell used

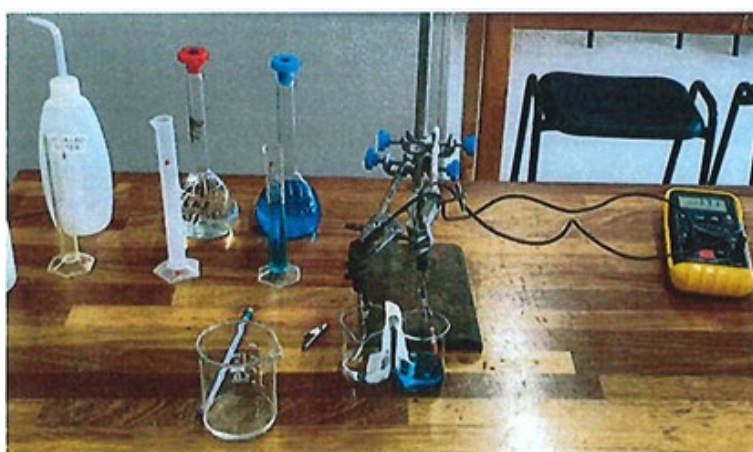


Figure 6.2: Finalised design with clamped electrodes to prevent change of the depth of the electrode

## Practical

### Chemicals

Label	Identity	Hazard information
Copper	Multiple strips	Not classified as hazardous in this
Magnesium	Multiple strips	Flammable
Copper Sulfate	1.0 mol dm <sup>-3</sup> 44.4g of copper sulfate	Eye irritant, environmental hazard
Magnesium Sulfate	0.4 mol dm <sup>-3</sup> 12.04g of magnesium sulfate	Eye irritant, environmental hazard
Potassium Nitrate	Saturated	Respiratory irritant

### Equipment

- 2 Beakers (100 ml)
- Large Beaker
- Multimeter
- Eye protection
- Gloves
- Measuring cylinder (25 cm<sup>3</sup>)
- Clamp Stand, base, clamp
- Crocodile Clips and Wiring
- Strips of Filter Paper
- Volumetric Flasks
- Digital Scale
- Conical Flask

### Health and Safety

- Wear eye protection throughout
- Wear gloves when handling potassium nitrate and salt bridge

### Part 1- Making a standard solution

1. Place the weighing boat on a digital scale and record its weight.
2. Add 44.4g of copper sulfate to the weighing boat.
3. Place the copper sulfate in the conical flask and fill with water.
4. Stir until dissolved adding more water if required
5. Transfer the solution into the volumetric flask using distilled water to rinse the conical flask; ensuring the whole solution is in the volumetric flask.
6. Insert the stopper and mix the contents by turning the flask upside down then upright multiple times.
7. Remove the stopper and fill the volumetric flask up to the 250 cm<sup>3</sup> line.
8. Repeat step 6.
9. Accurately label the flask.
10. Repeat the process using 12.04g of magnesium sulfate



## Part 2 - Making an electrochemical cell

1. Using a measuring cylinder measure out 25 cm<sup>3</sup> of magnesium sulfate and pour into a 100 cm<sup>3</sup> beaker.
2. Use a different measuring cylinder to pour out 25 cm<sup>3</sup> of copper sulfate into the other beaker.
3. Attach the magnesium and copper electrodes to crocodile clips. Clean the electrodes if needed by soaking them in a strong acid.
4. Wire these clips to a voltmeter.
5. Clamp the wires using a clamp stand to ensure that the effective depth of the electrodes is kept constant using extender wires if necessary. As shown in Figure 6.1
6. Place the beakers underneath the clamp in such a way that each metal can access their respective metal sulfate solutions.
7. Put each metal electrode in their respective metal solutions.
8. Pour a small amount of potassium nitrate into a beaker (<20ml).
9. Take a strip of filter paper and thoroughly soak it into the potassium nitrate solution.
10. Put one end of the paper into each beaker.
11. Record the voltage.
12. Repeat the experiment 10 times successively lowering the concentration of the solution used by 90%.
13. Plot a graph of voltage against concentration.

## Making a pH probe

1. Attach the magnesium and copper to crocodile clips.
2. Wire the crocodile clips to a voltmeter.
3. Pour out an acidic pH buffer with a low pH (<3) into a beaker.
4. Use a digital pH probe to record the pH of the buffer.
5. Place the electrodes into the solution and record the voltage
6. Repeat the experiment multiple times.
7. Serially dilute the buffer solution and repeat steps 1-6 to ensure a wide range of pH results are recorded.
8. Plot a graph of pH against voltage and draw a line of best fit.
9. Calculate the equation of the line.
10. Use this to calibrate your voltage readouts

## Results

Log[CuSO <sub>4</sub> ]	EMF (V)
0	1.837
-1	1.808
-2	1.774
-3	1.757
-4	1.715
-5	1.696
-6	1.694
-7	1.659
-8	1.631

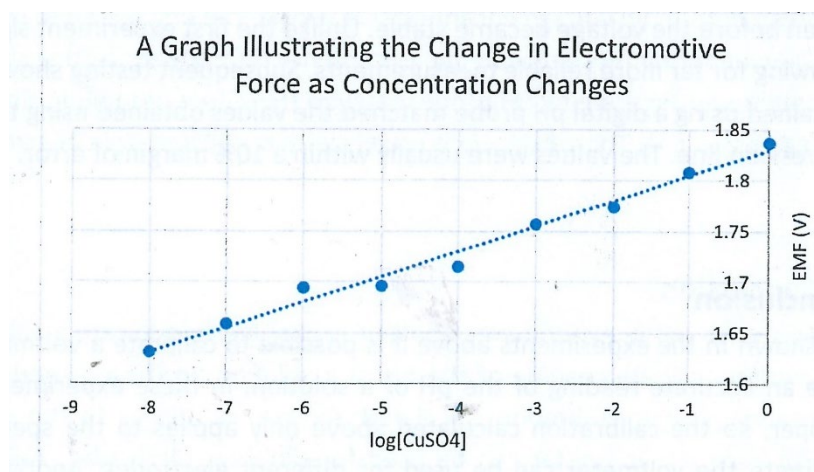


Figure 7.1: Table of results of part 2

Figure 7.2: Graph of data in figure 7.1

pH	Voltage reading (V)				
	1	2	3	4	Average
6.12	1.419	1.445	1.446	1.427	1.434
4.40	1.458	1.461	1.470	1.475	1.466
1.68	1.655	1.659	1.647	1.660	1.655
3.46	1.341	1.338	1.349	1.333	1.340
5.02	1.280	1.246	1.206	1.189	1.230
1.93	1.722	1.725	1.733	1.732	1.728
3.85	1.320	1.337	1.328	1.331	1.329

Figure 8.1: Table of results of part 3

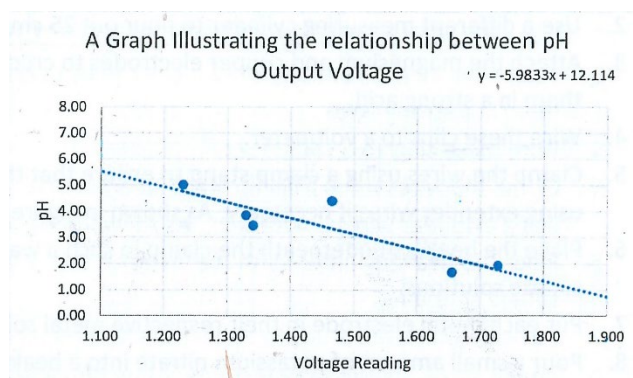


Figure 8.2: Graph of data in figure 8.1

## Analysis of Results

### Voltage on Concentration

Least square regression line:  $V = 0.0249(\log []) + 1.8296$

PMCC: 0.9918

The first experiment gave data that showed a clear logarithmic relationship between concentration and EMF however since there is only significant change when there is a difference of magnitudes of 10 the set-up wouldn't be suitable at differentiating smaller differences in concentration. The set-up would work to discern 0.1 MOL from 1 MOL but would be less reliable for concentrations such as 10 MOL and 9 MOL. Since the relationship is very strong the set-up would be very reliable to measure the smaller concentrations if more precise instruments were used.

### pH on Voltage

Least square regression line:  $\text{pH} = -5.9833V + 12.114$

PMCC: -0.8815

The second experiment gave data that showed a linear relationship between pH and EMF. The first value in figure

8.1 was anomalous so has been omitted from the graph and data calculations: this could be due to the reading being taken before the voltage became stable. Unlike the first experiment significant change occurs at much smaller scales allowing for far more reliable measurements. Subsequent testing showed that this was the case as the value we obtained using a digital pH probe matched the values obtained using the experimental set-up calibrated using the regression line. The values were usually within a 10% margin of error.

## Conclusion

As shown in the experiments above it is possible to calibrate a voltmeter with two electrodes of different metals to give an accurate reading of the pH of a solution. In these experiments the electrodes used were magnesium and copper, so the calibration calculated above only applies to the specific electrodes used but the method used to calibrate the voltmeter can be used for different electrodes. Another factor to be considered when calibrating a voltmeter in this manner is the area of the electrode in contact with the solution so to use our specific calibration the electrodes need to have the same surface area exposed to the solution as we did.

The calibrated voltmeter created from these experiments is essentially a pH probe which functions in a similar fashion. A pH probe works by having a glass electrode that contains alkali metal ions; the metal ions react with the freely moving  $\text{H}^+$  ions in the solution and produce a potential difference [sJ]. Using a pH probe to find the pH of a solution is more accurate than using an indicator that requires someone to subjectively judge the results through difference in colour.

As well as being a way to easily read the pH of a solution, part one of the experiment shows that the set-up can also be used to read the concentration of a solution. Unlike the linear relationship of voltage and pH, the relationship of concentration and voltage is logarithmic. This particular use of the set up would be beneficial for faster readings of concentrations of ionic solutions- given the electrode is composed of the same metal as the metal ions of the solution. If this set-up used more precise instruments, it could give accurate readings of concentration which would be a lot faster than having to carry out a full titration.

Overall, both experiments showed that there is a direct relationship between concentration and the electromotive force of an electrochemical cell, and that this relationship can be used to calibrate a voltmeter to be used as an accurate pH probe.

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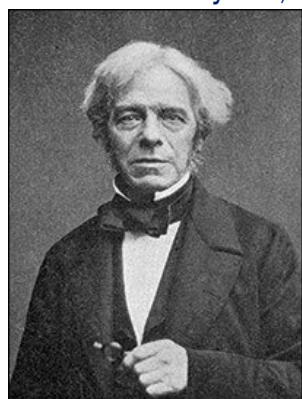
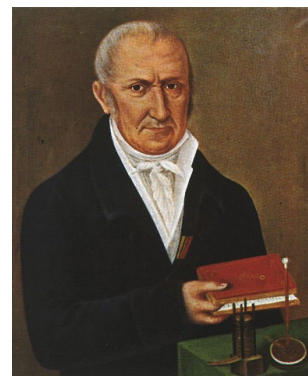
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## HISTORY OF ELECTROCHEMISTRY: FROM VOLTA TO FARADAY

The story of electrochemistry begins with **Alessandro Volta**, who announced his invention of a device that became known as the voltaic pile, the first modern electrical battery, in 1800. Volta had built his voltaic pile to challenge Luigi Galvani's claim to have demonstrated that animals produce electricity. According to Volta, Galvani's results came from his use of two dissimilar metals connected by a moist conductor (a frog's leg). Volta reproduced this configuration in his new invention, which consisted of pairs of zinc and silver disks connected by brine-soaked cardboard.

Galvani had incorrectly thought the source of his "animal electricity" was in the animal. Although Volta's work proved that this was not so, he himself did not fully understand how or why his voltaic cell arrangement produced electricity. He thought it must be in the physical properties of the two metal electrodes. But it took another great scientist and a few more years, the Englishman **Michael Faraday**, to solve the puzzle.



Some 30 years after Volta's invention, Faraday showed that the voltaic cell was actually chemical in nature. Faraday introduced new terminology to the language of chemistry: *electrode (cathode and anode)*, *electrolyte*, and *ion (cation and anion)*. Faraday correctly identified the source of electricity, or precisely *electromotive force (emf)*, as the chemical reactions at the two electrode-electrolyte interfaces. His two laws of electrochemistry, published in 1834, predict how much product results from passing a certain amount of current through a chemical compound or its solution, a process that he named "electrolysis."

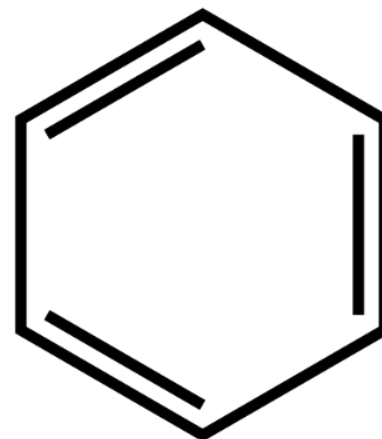
These laws are still fundamental to industrial electrolytic production of metals and other chemicals.

## **ARVIN GEORGE**

### **BENZENE: HERO or VILLAIN?**

#### **Discovery and History**

Benzene was first isolated in 1825 by Michael Faraday in illuminating gas. Consequent experiments showed that the molecule had an amount of carbon and hydrogen atoms ( $C_6H_6$ ). Benzene can occur if certain organic compounds are partially burned. It is commonly found in cigarette smoke, volcanoes, forest fires and as a by-product of some industrial processes. One of the first commercial uses of benzene was as an aftershave, however its carcinogenic properties would later discourage that particular use. Benzene also saw use as a solvent, degreaser and as an anti-knocking agent in petrol engines. Currently benzene is used as an important raw material in petrochemical manufacturing, polystyrene production and phenol production.



#### **Structure and properties<sup>[1]</sup>**

The structure of benzene was subject to much interest and debate in the 19<sup>th</sup> century. In 1861 Joseph Loschmidt proposed that benzene had a cyclical structure, composed of alternating carbon-carbon double bonds separated by single carbon bonds. A modified structural formula was proposed by August Kekule, in which the double bonds would oscillate producing two equivalent but different structures that would exist in rapid equilibrium. Finally in 1931 Linus Pauling proposed that benzene had a single structure, which was a resonance hybrid of the two structures proposed previously.

The stability of benzene can be attributed to its delocalised electrons. Since each electron is now attracted to all six carbons in the ring, the electrostatic force between them is now relatively stronger and they are now “held” closer, figuratively speaking.

The molecule is arranged in a planar structure with bond angles of  $120^\circ$  between carbons. It has a boiling point of 80.1 degrees Celsius and a melting point of 5.5 degrees Celsius.

#### **Controversy**

Benzene is often subject to controversy. This stems from numerous high profile incidents linked to its toxicity. For example, in 1990 a bottled water manufacturer, Perrier, was forced to recall their water as it was found to be contaminated with benzene.<sup>[2]</sup> Benzene has also leaked from underground storage tanks into wells and other water storage systems, causing further contamination. The high risk of anaemia and potential damage to the immune system is life threatening.

#### **Conclusion**

Benzene is highly toxic and should be treated with care, as potential leaks or contamination is very hazardous. We should not, however, discount its immense value to the petrochemical industry, especially its invaluable role in the production of polymers. In conclusion, as long as proper safety protocols are heeded, benzene's benefits outweigh its risks.

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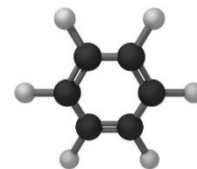
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## EMIL CHERIYAN

### IS BENZENE A HERO OR VILLAIN?

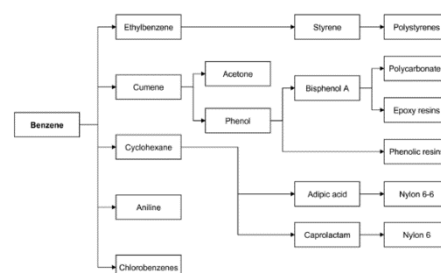
- Benzene was first introduced to the world by Michael Faraday, an English chemist, in 1825. Michael at this time was producing illuminating gas – he identified and isolated benzene from burnt whale oil.
- Later Eilhard Mitscherlich, in 1833, formed benzene through distilling benzoic acid and lime. He also discovered that benzene had the formula of  $C_6H_6$ .
- Then in 1845 Charles Mansfield, who was working under August Hoffmann, produced benzene from coal tar which sparked the first industrial production of benzene. Hoffmann also used the word “aromatic” to create a series under its family’s patterns.
- After that, most importantly in 1865, Friedrich Kekulé came onto the scene intriguingly through a dream. In his dream Kekulé sees a snake trying to eat its own tail which inspired the shape of benzene, a ring structure.



The word “benzene” is taken from “gum benzoin”, or benzoic resin, which was known to the European pharmacists in the 16<sup>th</sup> century. A hydrocarbon was extracted and had names like *benzin*, *benzol* and *benzene*.

Benzene is a hexagonal molecule consisting of 6 carbons and 6 hydrogens and, through X-ray diffraction it was discovered that it is a planar structure with the carbon-to-carbon bond distance halfway between a C=C (1.34Å) and a C-C (1.46Å), so 1.40Å. The intermediate is caused by one electron delocalised from each carbon which is then found in the overlapping *pi* orbitals above and below the carbon ring. This means it has 3 double and 3 single carbon bonds, each carbon having a hydrogen bonded to it.

The main use of benzene is as an intermediate to form other compounds like cyclohexane, phenol, acetone etc. It is also used as an additive to petrol to increase octane rating and reduce engine knocking and the likelihood of an explosion within the car.



#### So why can benzene be portrayed as a villain?

One reason is because benzene is classed as a carcinogen and so increases the risk of cancer; but it is also a big cause of bone failure. If benzene is eaten it may cause a burning feeling throughout the digestive tract, nausea, vomiting and pain. Inhalation of benzene may cause neurological effects, problems with the heart and lungs, coma and convulsions, and can cause redness and swelling if in contact with the skin. Benzene works by causing cells not to work correctly. For example, it can cause bone marrow not to produce enough red blood cells, which leads to anaemia. Also, it can damage the immune system by changing blood levels of antibodies and causing the loss of white blood cells, which can lower the body’s ability to fight infections and might even be life threatening. Currently there is no cure.

However, benzene is quite useful and is one of the most used chemicals in the world. At the moment it is very widely used in industries like plastics, foams, dyes, detergents, solvents, insecticides and pharmaceuticals, etc. So, in my opinion it is not a villain but a hero.

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## LEONEL JOSE

### IS BENZENE a HERO or VILLAIN?

Benzene is a hydrocarbon with molecular formula  $C_6H_6$ . It was first isolated by Michael Faraday in 1825, from a by-product of the production of 'portable gas'. Portable gas is made by dropping whale or fish oil into a hot furnace, and then compressing the gas which was made. He distilled the liquid formed from when the gas was pressurised. Benzene was manufactured from coal tar at an industrial scale from the mid-1800s.[1] Before it was known to be carcinogenic, it was used in aftershaves and used in the process of decaffeination of coffee. Currently it is a very important starting chemical to make other useful chemicals in industrial processes. [1][2]

Kekule was one of the first to suggest a structure for benzene, describing it as six carbon atoms bonded together in a ring with alternating double and single carbon-carbon bonds. In this way it would seem very similar to alkenes, and so you would expect benzene to react in a similar way and undergo substitution reactions, but it rarely does so. Instead, its hydrogen atoms are replaced by other atoms or groups atoms in substitution reactions. Carbon to carbon double bonds " and carbon to carbon single bonds have different lengths and so with Kekule's model, you would also expect the arrangement of carbon atoms to not form a regular hexagon, but this is actually the case, with the bond lengths between the carbons being exactly the same. One final discrepancy is that benzene is more stable than what Kekule's model suggests. When looking at the hydrogenation of benzene to cyclohexane, we see that its enthalpy change is  $-208\text{ kJ mol}^{-1}$ , but when considering the hydrogenation of cyclohexene to cyclohexane, we see that this value is much lower than what we would predict. The enthalpy change of the hydrogenation of cyclohexene to cyclohexane is  $-120\text{ kJ mol}^{-1}$ , and so we would expect the enthalpy change of the hydrogenation of cyclohexene to cyclohexane to be three times this value,  $-360\text{ kJ mol}^{-1}$ . [3][4].

This evidence led scientists to develop a new model for the structure of benzene: the delocalised model. This model considers the atomic orbitals of each of the carbon atoms. For each carbon atom, there are three  $sp^2$  orbitals and one p orbital. There is overlap of the p orbitals laterally for each of three double bonds, and a  $\pi$  system is formed. This system is made up of two clouds of electron density, above and below the plane of the molecule. This delocalisation means that all the carbon to carbon bonds are identical.[4]

While Benzene has been very useful to us, it is also carcinogenic, known to cause acute myeloid leukaemia/ acute non-myeloid leukaemia [1]. Most exposure is through inhalation and some sources of exposure include use of benzene containing consumer products, exposure to cigarette smoke and inside vehicles. Some acute health effects include headache, dizziness and irritation to respiratory tract.[5]

Overall, there are many advantages to benzene in the manufacture of other useful chemicals. However, while there are many uses for it, it is a carcinogenic chemical, but it can be handled in an appropriate way to avoid exposure to it. So while it is not a hero, it is not a villain either.

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## **THOMAS ROY**

### **IS BENZENE A HERO OR VILLAIN?**

Benzene was discovered in 1825 by Michael Faraday and in the early 20th century, and was used as an after-shave lotion due to its sweet odour. It is a colourless liquid at room temperature and can be found naturally in crude oil.

Benzene is a hydrocarbon with the molecular formula  $C_6H_6$ . It has 1 carbon atom for every hydrogen atom by having the six carbon atoms in a planar hexagonal ring and one hydrogen attached to each carbon atom. Each free electron from the carbons form *pi* bonds which overlap with the others and allow the electrons to have free movement around each carbon giving it the property of aromaticity.

#### **Uses of benzene**

- Used in the production of rubber. Many products that contain benzene is used as solvents in production to make plastics, resins, and nylon and synthetic fibres.
- Contained in products in the printing industry for cleaning and maintaining printing equipment.
- Used in manufacturing chemical synthetic products such as nylon and Styrofoam.
- Used as a solvent in the pharmaceutical industry and can be found in certain drugs like aspirin which is a widely used for pain relief.

#### **Dangers of Benzene**

Exposure to benzene can cause serious problems by damaging your DNA. Long-term exposure to benzene can increase the risk of developing leukaemia.

Benzene can cause bone marrow not to produce enough red blood cells which can lead to anemia which **is** a condition where you lack enough healthy red blood cells to carry adequate oxygen to your body's tissues.

It can weaken the immune system by changing blood levels of antibodies and causing the loss of white blood cells.

Short-term exposure can lead to drowsiness, dizziness, headaches and unconsciousness.

Eating foods containing high levels of benzene can cause vomiting, irritation of the stomach, sleepiness, and irregular heartbeat.

Overall, I think the benzene is closer to a villain than a hero due to all the risks of serious problems although I still think that it should be used as it is very useful in medicine and the production of many day-to-day things.

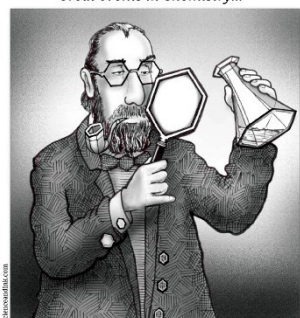
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Great events in Chemistry...



1865: Kekulé, moments before his brilliant insight into the structure of benzene.

**It is always good to maintain a healthy sense of humour, even in chemistry. Maybe even Friedrich Kekulé himself might have chuckled over this cartoon caricature of his great discovery.**

**If you don't "get it", have a closer look – the clue is in the glasses!**

## ***BERYL JOHN***

### **IS BENZENE a HERO or VILLAIN?**

Benzene is a villain, as it is quite dangerous to humans and animals. Although it is a very useful chemical in industry and can be used to produce many things such as plastic, fuel, detergents and solvents, there are many alternatives to benzene which aren't carcinogenic; for example, pentane, cyclopentane and diethyl ether.

#### **History**

Benzene was discovered in 1825 by Michael Faraday. Faraday isolated benzene from the oily residue that was created as a byproduct of portable gas production. He distilled the liquid that formed when the portable gas was pressurised, and separated benzene (which he called 'bi-carburet of hydrogen').

#### **Structure**

Benzene is an aromatic hydrocarbon with a molecular formula of  $C_6H_6$ , therefore each carbon atom is bonded to 2 other carbon atoms and 1 hydrogen atom. The bond angles in benzene are all equal (120 degrees), forming a planar hexagonal ring. Each carbon forms 3 sigma bonds and 1  $\pi$  bond. The p orbitals overlap above and below to form delocalised  $\pi$  bonds (or a delocalised system) above and below the plane of the molecule. All the bonds in benzene are of an equal length due to the 6 delocalised electrons which also account for the stability and low reactivity of benzene.

Cracking crude oil produces many different hydrocarbons, benzene being one of the basic feedstock petrochemicals. Benzene is an important starting material also used in producing many other chemicals, mainly fuels and plastics. It is used as an intermediate to produce many other organic compounds, such as ethyl benzene, cumene, cyclohexane and nitrobenzene. More than half of the entire benzene production is processed into ethyl benzene, a vital intermediate in the production of styrene, which is used to make polystyrene. Benzene was previously used in aftershaves and decaffeinating coffee but now it has been banned.

#### **Dangers**

The highest levels of exposure to benzene are usually in industrial settings, however the most common exposure to benzene is from air pollution or cigarette smoke. The average amount of benzene found in tobacco smoke is approximately 35- 80 ug for someone who smokes 40 cigarettes in a day, they would be taking in up to 3 mg a day of benzene. Not only is this dangerous to smokers, but also to those who passive smoke. In humans, benzene is a Group 1 carcinogen\*, causing damage to the immune system and preventing cells from functioning normally.

Benzene can be found from oil spills and industrial waste, however measures such as new legislations and catalytic converters have reduced the amount of benzene that is being emitted. High levels of benzene exposure can result in benzene poisoning, excessive bleeding, anaemia or even death.

\*The IARC classifies substances into groups based on how carcinogenic they are. Group 1 is used to describe substances that are proved to be carcinogenic in humans/experimental animals.

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One of Michael Faraday's original benzene samples is still here today – it is an exhibit in the Faraday Museum in the basement of the Royal Institution building in London.

A small vial containing Faraday's sample of benzene, a colourless liquid with a sweet smell, is on display. The vial is contained within a larger glass bottle for safety and,

if you look closely, you can see the words '*Bicarburet of hydrogen*' scratched onto both the vial and the bottle. This would have been done by Faraday himself.

Benzene is a natural hydrocarbon and a component of crude oil. Faraday isolated this substance for the first time in 1825 while investigating an oily residue that was created as a by-product of the production of 'portable gas'. Portable gas was created by dropping whale or fish oil into a hot furnace, compressing the gas which was created and then storing it in containers to use in lamps in private and public buildings.

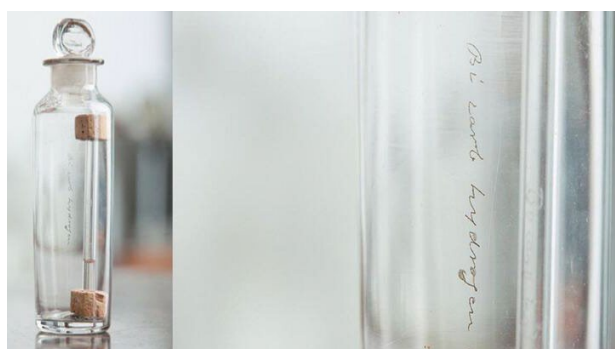
Faraday was interested in the liquid which formed when the gas was pressurised. He experimented with distilling this mixture and, as he reported in a paper to the Royal Society on 16th June 1825, he '*succeeded in separating a new compound of carbon and hydrogen, which I may by anticipation distinguish as bi-carburet of hydrogen*'.



The structure of benzene has been the subject of much research. Many theories were put forward throughout the 19th century but it wasn't until 1925, a century after its discovery, that **KATHLEEN LONSDALE** confirmed the structure through X-ray crystallography, using photos taken at the Royal Institution (although not from this sample).

Kathleen Lonsdale was a 22 year old graduate scientist at the time and would go on to play a fundamental role in establishing the new science of X-ray crystallography. During her time at the RI Lonsdale worked in many areas related to X-ray crystallography - both theoretical and experimental. She was awarded a DSc by University College London in 1936 and in 1945 she and Marjory Stephenson became the first women Fellows of the Royal Society.

She became professor of chemistry at University College London in 1949 and, in 1956, she was created a Dame of the British Empire.



## **CLARA SAIU-BELL**

### **BENZENE – HERO OR VILLAIN?**

Despite Benzene being used for a wide variety of industrial processes, it does not outweigh the countless harmful effects that have been proven by research on humans and animals.

#### **History**

Michael Faraday discovered Benzene in 1825 when he isolated it from oily residue which was produced by compressed illuminating gas, made from whale or fish oil.

#### **Structure**

Benzene is the simplest aromatic hydrocarbon. With a molecular formula of  $C_6H_6$ , it is composed of six carbons in ring, in which every C-C bond has a length that is halfway between a single and double bond. Each carbon has one hydrogen atom attached to it ( $C_6H_6$ ) and one delocalised electron where the  $\pi$  orbitals overlap.

Unsaturated hydrocarbons are usually more reactive and less stable than Benzene is. This is because the charge of these delocalised electrons in Benzene is spread equally across the whole molecule, meaning they are held more strongly.

#### **Uses**

Previously, Benzene was used in domestic solvents, cosmetics, and decaffeination of coffee. However, this is no longer allowed now that the dangers have been discovered.

Benzene is used as a building block for many chemicals that are used in industrial manufacturing processes, including production of plastics, dyes, lubricants, detergents, solvents, drugs, pesticides and even explosives.

#### **Dangers**

Benzene is released into our environment through the evaporation and manufacture of petrol. It can also be released by industrial waste and spills. Exposure to benzene can cause irritation in the throat, nose and eyes, as well as coughing and breathing difficulties. Higher levels of benzene in the environment can lead to airways swelling and fluid building up in the lungs. It has also been proven that ingesting or inhaling benzene can have negative neurological effects.

Research has linked Benzene to myeloid leukaemia, non-lymphocytic leukaemia, and potentially other types of cancer. It is classified as carcinogenic to humans.

It is because of these risks that benzene levels are strictly controlled to limit exposure in our environment, and therefore reduce health issues.

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## **SAMUEL EVANS**

### **BENZENE**

**C<sub>6</sub>H<sub>6</sub>** - simplest of the aromatic hydrocarbons; clear, flammable liquid

- Aromatic rings have alternating carbon single and double bonds
- Carbon atoms in a hexagonal ring
- Benzene occurs naturally when some organic compounds are partially burned
- Present in cigarette smoke
- First produced industrially as a by-product of making clean burning coke from coal

#### **Hero or Villain?**

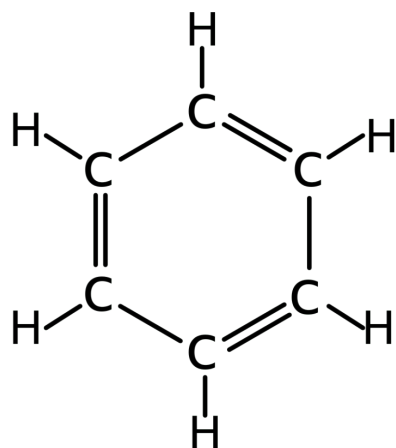
One of the first uses of benzene was as an after-shave; however, it is now known to be a powerful carcinogen – a substance that promotes the formation of cancer.

Benzene is used as a solvent, degreaser and an anti-knocking agent in petrol.

Benzene is now known as an important raw material in petrochemical production, including the production of styrene.

Styrene is notably used to make many plastic polymers, such as polystyrene.

Benzene is often given a bad reputation as a toxic substance that causes much harm to the environment.



Molecular weight: 78 g mole<sup>-1</sup>

Benzene can have quite a detrimental effect on human health as it is toxic and a powerful carcinogen. On top of this, it is a highly flammable liquid and many products produced with the use of benzene must be handled carefully.

Despite all this I believe benzene is a hero. It has played an essential role in manufacturing. It is used continually and ranks in the top 20 in production volume in the United States for chemicals. It is used to make a plethora of other chemicals, as well as being present in many important products.

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It may come as a surprise that benzene is present in tobacco smoke, but it is actually one of about 7,000 chemicals that are produced when you light up a cigarette, cigar or pipe. Some 250 are really nasty, such as hydrogen cyanide, carbon monoxide and ammonia. 69 are known carcinogens, including benzene.

It may also come as a surprise that smoking does not just cause lung or throat cancer. At least 14 different types of cancer are caused by smoking, including the oesophagus, stomach, kidney, bowel and bladder. Smoking also damages your heart and your blood circulation, increasing your risk of developing coronary heart disease and cerebrovascular disease.



**Simple advice: do not start smoking; and if you are already a smoker – give up!**

## KISHORA SUTHAKARAN

### BENZENE

Benzene was first discovered by the English scientist Michael Faraday in 1825. It was a molecule that stood wonders towards the science community. Benzene can produce a variety of material and ultimately consumer goods. Benzene has many benefits to it as it is used in many resources as it is used in lubricants, rubbers, dyes and detergents, drugs and explosives and pesticides. Benzene has many disadvantages as well as its benefits as different levels of poisoning arise depending on the amount, route and the length of time, exposure, and medical condition of a person. Small amount of benzene cannot cause much damage however breathing large amounts of it can cause serious health issues. The website “Chemical safety facts.org” highlighted some of the key dangers of benzene and somewhat justifies why it is so evil. It recalls benzene as causing symptoms such as drowsiness, dizziness and unconsciousness. Long term benzene exposure causes effects on bone marrow which can cause anaemia and leukaemia.

#### Structure of benzene

Benzene is a planar hexagonal molecule made up of six carbon atoms and six hydrogen atoms and has a molecular formula of  $C_6H_6$ . It has a bond angle of 120 degrees. Its structure explains why this molecule is very unreactive. Benzene's structures give rise to some of its properties including:

- *Delocalized electrons above and below the plane of ring*
- *Presence of delocalized electrons makes benzene particularly stable*
- *Benzene resists addition reactions because those reactions would involve breaking the delocalization (chem.libretexts)*

*Benzene in itself overall has important features that help us economically with our crops and it is an important molecule in our modern-day life. However, it can cause many other dangerous problems which can cause detrimental effects on humans and their existence such as cancer, so it can secretly show its advantageous side but really inside our bodies it is the true villain.*

#### Source of information

<https://www.britannica.com/science/benzene>

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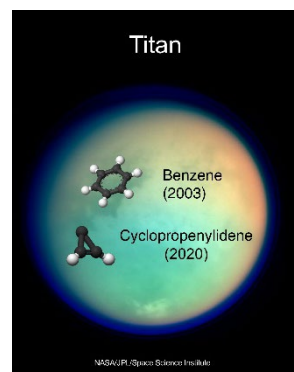
**BENZENE IN SPACE!** Benzene is not just found here on Earth. Wherever we look in the Universe we find its trace – in other atmospheres, around other stars, even in interstellar clouds. This is quite exciting because it might give us a clue that there is the possibility of life throughout the cosmos, not just here.

Life as we know it is based on the ability of the carbon atom to form ring-shaped molecules so, when it was shown during the 20<sup>th</sup> century that short chain hydrocarbons, such as methane and ethane, were present in interstellar clouds, the search was on for the first “ringed molecule”, benzene. Finally, in 2001, a Spanish team of astronomers that observed with ESA's Infrared Space Observatory (ISO) reported the first detection in interstellar space of benzene, the ring molecule par excellence.

It is thought that benzene is produced by stars at a specific stage of evolution, and that it is an essential chemical step towards the synthesis of more complex organic molecules whose true nature is still unclear -although their fingerprints are now very conspicuous in the Universe.

In our own Solar System, perhaps the most exciting discovery of benzene was in the atmosphere of Saturn's largest moon, Titan. Unusually for a moon, Titan has retained a comparatively thick atmosphere, about 60% Earth's atmospheric pressure, but it is mainly nitrogen (about 95%) and methane (about 3-5%), with the rest being a mixture of more complex hydrocarbons, including benzene and an even more interesting molecule, cyclopropylidene!

<https://sci.esa.int/web/iso/-/25880-iso-detects-benzene-in-space>





## **ADRIAN SMITH-DELGADO**

### **BENZENE**

#### **Introduction**

By the end of 2022, global production capacity of Benzene is set to rise to 71 million metric tons, a 16.4% increase from 2017.

Benzene is perhaps one of the most important chemical species known to man; its versatility and ease-of-commercial-production have certainly helped it to rise to and consolidate its role as a mainstream chemical.

However, as history has shown: no “wonder” chemical is free of scrutiny and flaws, benzene being no exception ...

#### **A Brief History**

Benzene’s history is an excellent example of the struggles faced by 19<sup>th</sup> century chemists, in a period where the discipline as a whole was advancing at an extraordinary pace.

- 1825, saw Michael Faraday isolate Benzene from an oily film which had formed from a specimen of “lighting” gas-he was quick to notice that the molecule had an equal number of carbons as it did hydrogens, and hence named the molecule “carbureted hydrogen”.
- 1834, saw German chemist Eilhard Mitscherlich extract benzene from a specimen of gum benzoin, which prompted him to propose the name “benzin” – however this name was not popular due to the ambiguity surrounding its similarity to alkaloids. He later suggested the name benzol [derived from the German word for oil “öl”]; however, this name was changed to benzene to avoid misidentification of benzene as an alcohol.

While the existence of Benzene was apparent to chemists like Faraday as early as 1825, its structure would not become immediately apparent until almost half a century later ....

[See next section]

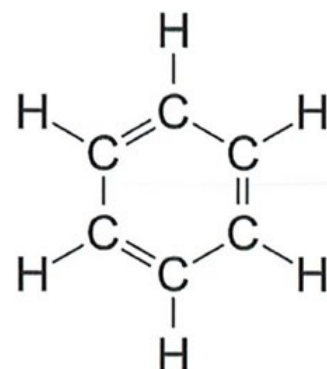
#### **Structure and Properties**

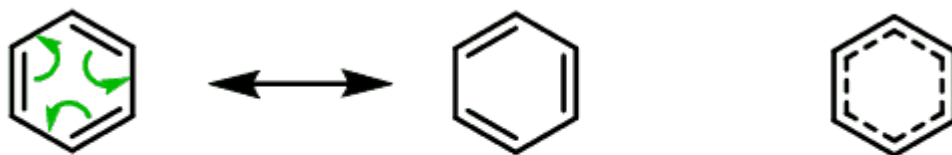
Perhaps the most notable feature of benzene from a structural standpoint, is the equal number of carbon and hydrogen atoms:  $C_6H_6$  a direct consequence of 3 double bonds and 3 single bonds [which alternate such that 2 double bonds are not adjacent].

The molecule is an aromatic hexagonal planar one, with bond angles of 120 degrees. (Shown on right)

However, this is relatively simplistic. Structural analysis of Benzene does little to explain the chemical’s properties. Hence we must explore such properties further.

Perhaps the most extraordinary property of benzene, is its reactivity – or lack of it. For an organic molecule with 3 double bonds, this is an unexpected property [to say the least], as areas of such high electron density typically react quickly with molecules involving both permanent and temporary dipoles – however benzene will not react with halides such as bromine, without the introduction of electromagnetic radiation and/or catalysts: hence we can postulate that there must be something unique about the C-C and C=C bonds in benzene. The reason for such a unique property can be attributed to and explained by the phenomenon of resonance structures/effect:





## Resonance structures

## Hybrid

- The structure of Benzene was highly disputed, however German chemist August Kekule proposed two structures: they differed only in the position of the three double bonds.
- If we find an equilibrium between both Kekule structures, we end up with an intermediary [referred to as a resonance hybrid] with 1.5 bonds between any two adjacent carbon bonds [1 Sigma bond & 1/2 a Pi Bond].
- The lack of a complete pi bond explains why benzene cannot react with bromine.

### Contemporary Usage

Benzene is currently used in the following processes/industries:

- Benzene is widely used in the motor fuel industry, where it is used as a substitute for lead - thus simultaneously reducing the damage to internal combustion engines caused by "engine knocking" [where the fuel in an internal combustion engine prematurely ignites] and removing the dangers to health caused by lead
- Paint production: Benzene's relatively low price per volume allows it to be readily obtained and used as the primary solvent in paints - thus allowing for higher quality paint that is inexpensive to produce
- Used in detergents, in the form of benzene-based surfactants - chemicals that help to remove stains by lowering the surface tension between water and particulates
- Used to produce ethyl benzene, which is used to produce styrene and hence polystyrene, which are both used in the manufacture of plastics and synthetic rubber
- Used to produce phenol, which is used in the production/formulation of painkillers, explosives, dyes and mild antiseptics.

### Advantages and Disadvantages

#### Advantages

- Highly Versatile-Has numerous useful derivatives
- General lack of reactivity makes it significantly safer to handle
- Relatively cheap to produce on a commercial scale
- Widely available

#### Disadvantages

- Known to be carcinogenic, classified as such by the IARC [Institute Agency for Research on Cancer]. Can cause acute myeloid leukaemia and acute non-lymphocytic leukaemia
- Most benzene is produced commercially from the petrochemical industry and, to a much smaller extent, the coal industry – both of which utilize non-renewable sources
- Known air pollutant, which often reacts with other molecules in the atmosphere to form "smog" [which theoretically can be decomposed into less harmful products] which can then enter the water-cycle and hence contaminate water systems and soil

## Conclusion

To conclude: it would be difficult to accuse Benzene of being completely “good” or “bad”, due to the role it currently fulfills.

Benzene is extremely useful, and main industries rely on the species to produce consumer goods without which our lives would vary significantly.

However, Benzene cannot be relied on indefinitely due to the non-renewable nature of its major sources – its effect on the environment is gradually becoming increasingly pronounced, and its effects on human health are alarming.

Therefore, it is clear that currently Benzene is perhaps the best molecule of its “kind”, however the discipline of chemistry must begin considering alternatives that are just as effective but also safer and better for the environment - such a process should be no different than the struggles faced by the chemists of the 19<sup>th</sup> century.

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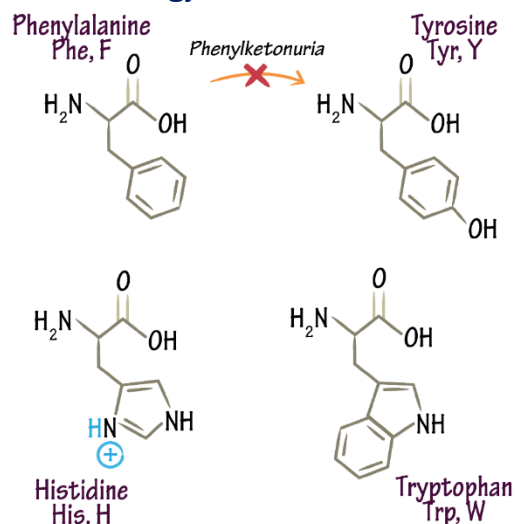
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## BIOLOGICALLY IMPORTANT COMPOUNDS WITH BENZENE RINGS

Although pure benzene might be more of a villain than a hero, it is not just in the field of industrial chemistry that the “benzene ring” is important. It is also vital in natural biology.....

Substances containing the benzene ring are common in both animals and plants, although they are more abundant in the latter. Plants can actually synthesise the benzene ring themselves from carbon dioxide, water, and inorganic materials. Animals cannot synthesise it, but they are dependent on certain aromatic compounds for survival and therefore must obtain them from food. Phenylalanine, histidine, tyrosine, and tryptophan (essential amino acids) and vitamins K, B2 (riboflavin), and B9 (folic acid) all contain the benzene ring. Many important drugs, such as aspirin, paracetamol and ibuprofen, also feature a benzene ring.

So perhaps we should cut benzene some slack and accept that it plays an important part of the most amazing phenomenon that we know...life itself!







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